Name Solution Key

CMSC 203 - Homework Assignment 4 - Due May 11, 2011

1. Consider the Sample Space of outcomes when a fair coin is tossed 6 times with an each outcome either a Head (H) or a Tail (T).

(a) What is the probability of the event of 4 Heads?

Let E = { 4 Heads } so |E| = C(6, 4) = 6! / 4!2! = 30/2 = 15 and S = { 6 coin tosses } so

$P(E) = |E| / |S| = 15 / 2^6 = 15 / 64.$

(b) What is the probability of the event of 4 Heads given the first toss is a Tail?

Let E = { 4 Heads } and F = { Txxxxx } and $|E \cap F| = |\{$ 4H in 5 tosses $\}| = C(5, 4) = 5.$

Hence, $P(E \mid F) = |E \cap F| / |F| = 5 / 2^5 = 5 / 32.$

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2. In relation to question 1, determine whether or not the probability of tossing 4 Heads is independent of the first toss being a Tail.

Denoting E = { 4 Head of 6 coin tosses} and F = { 6 coin tosses with first toss being a Tail }, from Question 1, we know P(E) = 15/64, P(F) = 1/2 and P(E|F) = 5/32.

If E is Independent of F, then P(E|F) = P(E), so we should have 5/32 = 15/64.
However, 5/32 = 10/64, therefore E is not Independent of F.

Alternatively, using the N-S-E-W test, we see that N = 5, W = 32 - 5 = 27, E = 10,
and S = 64 - 37 = 27, so E is Independent of F if NS = EW. Now, NS = 5(27) = 135,
and EW = 10(27) = 270. Since $135 \neq 270$, we see that E is not Independent of F.

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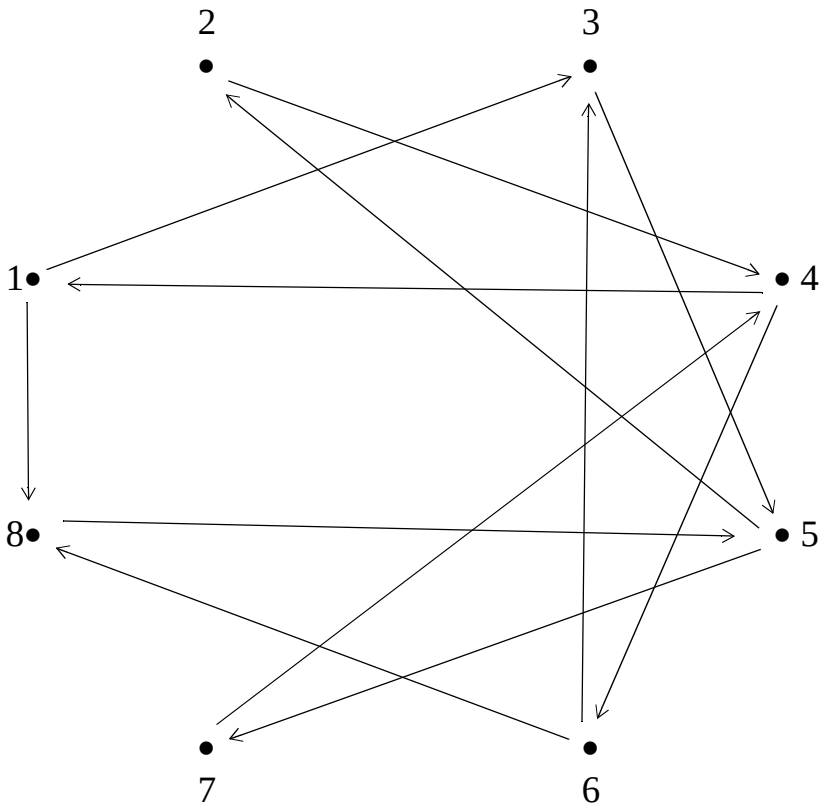
3. Draw the directed graph of the relation R on A = {1, 2, 3, 4, 5, 6, 7, 8} defined as
R = {(*a*,*b*) | *a*,*b* ∈ A and (*a* + 2) ≡ *b* mod 5}.



a	1	2	3	4	5	6	7	8
a + 2	3	4	5	6	7	8	9	10
b	3, 8	4	5	1, 6	2, 7	3, 8	4	5

R = {(1, 3), (1, 8), (2, 4), (3, 5), (4, 1), (4, 6), (5, 2), (5, 7), (6, 3), (6, 8), (7, 4), (8, 5)}

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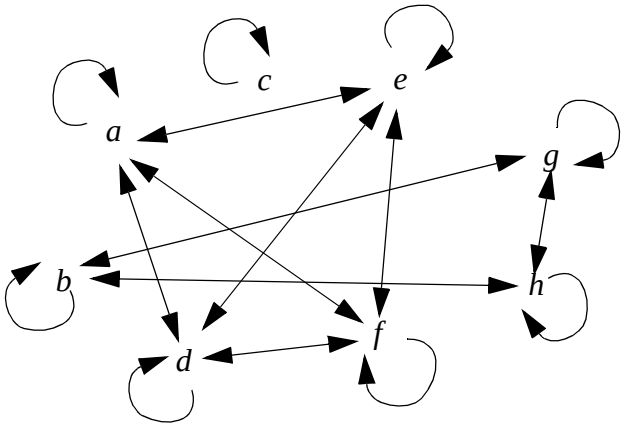
4. Consider the relation, R, on the set A = {*a, b, c, d, e, f, g, h*} given by the graph:



(a) Find [*e*]

[*e*] = { *a, e, d, f* }

(b) Find the partition of A induced by R

Partition(A) = { { *a, e, d, f* }, { *c* }, { *b, g, h* } }

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5. Let F be a function on the integers given by $F(n) = (n - 5)^2$.
(a) Show that the relation R = {(x,y) | x,y are integers and $F(x) = F(y)$}is a Reflexive, Symmetric, and Transitive relation.

Reflexive: If x is an Integer, then $x = x$, so $(x - 5) = (x - 5)$ and $(x - 5)^2 = (x - 5)^2$. Thus, (x, x) is in R, so R is Reflexive.

Symmetric: Let x and y be Integers with (x, y) in R. This implies that $(x - 5)^2 = (y - 5)^2$, so it follows that $(y - 5)^2 = (x - 5)^2$. Consequently (y, x) is in R, allowing us to conclude R is Symmetric.

Transitive : Let x, y, and z be Integers with (x, y) in R and (y, x) in R. This means:
$(x - 5)^2 = (y - 5)^2$, and $(y - 5)^2 = (z - 5)^2$, thus $(x - 5)^2 = (z - 5)^2$, so (x, z) is in R. Therefore, R is Transitive.

(b) Describe the partition of the integers induced by R.
Partition(**Z**) = { {5}, {4, 6}, {3, 7}, {2, 8}, {1, 9}, {0, 10}, {–1, 11}, {– 2, 12}, ... }

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6. Consider the database consisting of the following Fields and Records:

First Name	Last Name	Age	Phone	Height (in.)	Weight
Alan	Jones	26	555-1234	68	155
Mary	Smith	32	555-4321	65	128
Ted	Green	32	555-6789	74	210
Susan	Green	30	555-6789	69	144
William	Peters	26	555-9876	73	195
Peter	Williams	44	555-2468	69	185

(a) For this database, which Fields would serve as Primary Keys?

First Name and Weight are the Primary Keys.

(b) Find $P_{2,4}$

$P_{2,4}$ = { (Jones, 555-1234), (Smith, 555-4321), (Green, 555-6789), (Green, 555-6789), (Peters, 555-9876), (Williams, 555-2468) }

= { (Jones, 555-1234), (Smith, 555-4321), (Green, 555-6789), (Peters, 555-9876), (Williams, 555-2468) }