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## CMSC 203 - Homework Assignment 3 - Due April 20, 2011

1. Find the terms  $a_3$ ,  $a_4$ ,  $a_5$ , and  $a_6$  for the recursively defined sequence given by:  $a_0 = -1$ ,  $a_1 = 0$ ,  $a_2 = 1$ , and  $a_n = 2(a_{n-1})(a_{n-3}) + (a_{n-2})^2$  for  $n \ge 3$ .

2. In the study of Coding Theory, we quickly encounter recursions on binary strings of data. In particular, the notion of a Linear Recursive Sequence (LRS) is very useful. Consider the following binary sequence definition:

- 1. Start with initial fill: 100100100.
- 2. For the bits in positions  $x^9$  and  $x^3$ , calculate the bit in position  $x^0$  as  $(x^9 \oplus x^3)$ .
- 3. Shift one bit to the right and continue.



So, applying this polynomial to the initial "fill" 100100100 yields the next bit  $1 \oplus 1 = 0$ . Shifting the "taps" of the polynomial one position to the right and continuing, we get successive bits: 0, 0, 1, etc.

(a) Starting with the initial fill 1101000, apply the polynomial  $x^7 \oplus x^2 \oplus x^0$  to generate the next 4 bits.

(b) Starting with the initial fill 1111, apply the polynomial  $x^4 \oplus x^3 \oplus x^0$  until it "cycles".

3. Using Mathematical Induction, prove for all Natural Numbers n,  $\sum$ 

$$\sum_{i=0}^{n} i^{3} = \frac{n^{2}(n+1)^{2}}{4}.$$

4. Using the Strong Form of Mathematical Induction, prove for all Natural Numbers  $n \ge 3$ : If  $a_n = a_{n-1} + 2a_{n-2} + 3a_{n-3}$  and  $a_0 = 0$ ,  $a_1 = 3$  and  $a_2 = 6$ , then 3 divides  $a_n$ .

5. Suppose I have 30 different math books, 20 different chemistry books and 10 different biology books.

(a) How many ways can I line all these books on shelf that is large enough to hold them all?

(b) How many ways can I line them up so that all the books of the same type are grouped together?

(c) How many ways can I choose 3 of each to carry in a bookbag?

(d) How many ways can I perform (c) if I do not want to have a certain pair of biology books together in any of the selections?

6. The Mars Candy Company sells bags of M&Ms with 150 pieces candy colored from 10 different colors.

(a) How many different bags can they produce?

(b) How many different bags can they produce if each bag must contain at least 10 of each color?