Name __________Solution Key_______________

CMSC 203 - Homework Assignment 2 - Due March 30, 2011

1. For the function $f : \mathbf{R} \rightarrow \mathbf{R}$ defined as $f(x) = x^3 + 2$, show:

(a) f is One-To-One

Show that if $f(a) = f(b)$, then $a = b$, for any a and b in **R**.

Now, $f(a) = f(b)$ implies $a^3 + 2 = b^3 + 2$, so $a^3 = b^3$, thus $a = b$, for any a and b in **R**.

Therefore, f is One-To-One.

(b) f is Onto

Show for any y in **R**, there is a corresponding x in **R** such that $f(x) = y$.

Now, let y be in **R**. This implies $(y - 2)$ is in **R**, yielding that $(y - 2)^{1/3}$ is in **R**. Denoting $x = (y - 2)^{1/3}$, we see that $f(x) = [(y - 2)^{1/3}]^3 + 2 = (y - 2) + 2 = y$.

Therefore, f is Onto.

Name <u>Solution Key</u>

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2. What is the complexity of the following procedure?

```
PROCEDURE FOO(N: Integer)
NSQUARE = N*N
NCUBED = NSQUARE*N
COUNT = 1
OUT = 0
FOR I = 1 TO NSQUARE
    FOR J = 1 TO NCUBED
        OUT = OUT + I + J
        REMAIN = OUT MOD 2
        IF (REMAIN = 0) THEN COUNT = COUNT + 1
    NEXT J
NEXT I
OUTPUT(OUT, COUNT)
```

<u>Complexity</u>	<u>Code</u>
	PROCEDURE FOO(N: Integer)
1	NSQUARE = N*N
1	NCUBED = NSQUARE*N
1	COUNT = 1
1	OUT = 0
N^2	FOR I = 1 TO NSQUARE
N^3	FOR J = 1 TO NCUBED
1	OUT = OUT + I + J
1	REMAIN = OUT MOD 2
1	IF (REMAIN = 0) THEN COUNT = COUNT + 1
-	NEXT J
-	NEXT I
1	OUTPUT(OUT, COUNT)

This yields a total complexity of $4 + N^2(N^3(3)) + 1 = 3N^5 + 5$. Equivalently, the procedure is $O(N^5)$.

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3. Find the polynomial big-O estimate for the function: $(n^7\log^2 n + n^{11})(n^3 + 3n\log^2 n)$.

$$
\begin{aligned}
(n^7\log^2 n + n^{11})(n^3 + 3n\log^2 n) &\leq [(n^7(n^2) + n^{11}][n^3 + 3n(n^2)] \\
&= (n^9 + n^{11})(n^3 + 3n^3) \\
&\leq (n^{11} + n^{11})(4n^3) \\
&= (2n^{11})(4n^3) \\
&= 8n^{14}
\end{aligned}
$$

Therefore, $(n^7\log^2 n + n^{11})(n^3 + 3n\log^2 n)$ is $O(n^{14})$.

Name __________**Solution Key**_______________

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4. Use the Euclidean Algorithm to find GCD(2140, 136)

By the Quotient-Remainder Theorem, we have:

2140 = 136(15) + 100

and 136 = 100(1) + 36

and 100 = 36(2) + 28

and 36 = 28(1) + 8

and 28 = 8(3) + 4

and 8 = 4(2) + 0.

Thus, GCD(2140, 136)

= GCD(136, 100)

= GCD(100, 36)

= GCD(36, 28)

= GCD(28, 8)

= GCD(8, 4)

= GCD(4, 0)

= 4

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5. Using the Lemma below, prove that if two Integers divide each other, then they are equal.

Lemma: If the product of two Integers is 1, then the Integers each equal 1.

Prove: If A and B are Integers such that A divides B and B divides A, then A = B and B = A.

Proof: Let A and B be Integers such that A divides B and B divides A. Thus, there are Integers m and n with B = mA and A = nB.

Now, A = nB implies A = n(mA) = (nm)A. Thus, nm = 1. However, by the Lemma, we conclude it is the case that m = n = 1.

Therefore, A = B and B = A. *QED*

Name ___________Solution Key________________

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6. Using the following Lemma, prove $\sqrt{3}$ is irrational.

Lemma: If n is an Integer and 3 divides n^2, then 3 divides n.

Prove: $\sqrt{3}$ is irrational.

Proof: (Contradiction) Assume $\sqrt{3}$ is rational. This means, there exist Integers, p and q, such that prove $\sqrt{3} = p/q$, with $q \neq 0$, and such that p and q have no common factors.

Now, $\sqrt{3} = p/q$, implies $3 = p^2 / q^2$, so $p^2 = 3q^2$. This implies 3 divides p^2, hence the Lemma above allows us to conclude that 3 divides p. In that case, we have that there is an Integer, n, such that $p = 3n$, so $p^2 = 9n^2$.

Combining this with our previous observation, we have: $p^2 = 9n^2 = 3q^2$, thus $q^2 = 3n^2$. Again, this means that 3 divides q^2, so by the Lemma, we see that 3 divides q. However, this is a contradiction, since we have shown that 3 divides p and 3 divides q, so 3 is a common factor of p and q, but p and q have no common factors.

Therefore $\sqrt{3}$ is irrational. *QED*