CMSC 203 Spring 2011 Examination 3 Name______Solution Key_______

SHOW ALL WORK!

1. (6 points) How many license plates can a state produce if the plates can contain 7 characters (from 26 letters and 10 digits) if a certain pair of letters cannot be adjacent to one another and all the characters must be distinct?

ALL - (Plates with the pair as XY) - (Plates with the pair as YX)

= P(36,7) - P(35,6) - P (35,6) = (36! / 29!) - 2(35! / 28!)

2. (6 points) How many ways can a teacher choose 10 students from a class of 13 Boys and 16 Girls, if she must choose the same number of boys and girls?

(Choose 5 boys) AND (Choose 5 girls) = C(13,5)C(16,5) = (13! / 8!5!)(16! / 11!5!) or (13!16!)/(8!5!11!5!)

3. (6 points) How many orderings are there of the letters of the word *ELECTRICALENGINEERING* ?

ELECTRICALENGINEERING = EEEEELLCCTRRIIIANNNGG

Orderings = 21! / (5!2!2!2!3!3!2!) or C(21,5)C(16,2)C(14,2)C(12,1)C(11,2)C(9,3)C(6,1)C(5,3)C(2,2)

4. (6 points) How many ways can I seat 12 people around a circular table?

Orderings = (12 - 1)! = 11!

5. (6 points) How many ways can I fill a box of 50 chocolates from 15 types if I must have at least 2 of each type in the box?

Slots = 50, Categories = 15, Transitions = 15 - 1 = 14, Total Restrictions = 2(15) = 30 so

Total Boxes = C(50 + 14 - 30 , 50 - 30) = C(34, 20) = C(34, 14) = 34! / 20!14!

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Let R be the relation on **Z** given by R = $\{(a,b) \mid a,b \in \mathbf{Z}$ and $a \equiv b \bmod 10\}$.

6. (6 points) Show the R is Reflexive.

Let x be an Integer, then $(x - x) = 0 = 10(0)$, and 0 is an Integer, hence (x, x) is in R.

Therefore, R is Reflexive.

7. (6 points) Show the R is Symmetric.

Let x and y be Integers with (x, y) in R. This implies $(x - y) = 10k$, for some Integer k.

Thus, $(y - x) = -(x - y) = -10k = 10(-k)$. Since k is an Integer, $(-k)$ is an Integer, hence (y, x) is in R.

Therefore, R is Symmetric.

8. (6 points) Show the R is Transitive.

Let x, y, and z be Integers with (x, y) and (y, z) in R. Thus $(x - y) = 10k$ and $(y - z) = 10m$ for some Integers k and m.

This yields $(x - z) = (x - y) + (y - z) = 10k + 10m = 10(k + m)$. Since k and m are Integers, $(k + m)$ is also, hence (x, z) is in R.

Therefore, R is Transitive.

9. (6 points) Describe the partition of **Z** induced by R.

Partition(**Z**) = { [0], [1], [2], [3], [4], [5], [6], [7], [8], [9] }
= { {...,–20, –10, 0, 10, 20,...}, {...–19, –9, 1, 11, 21, ...}, {..., –18, –8, 2, 12, 22,...},
{..., –17, –7, 3, 13, 23,...}, {..., –16, –6, 4, 14, 24,...}, {..., –15, –5, 5, 15, 25,...},
{..., –14, –4, 6, 16, 26,...}, {..., –13, –3, 7, 17, 27,...}, {..., –12, –2, 8, 18, 28,...},
{..., –11, –1, 9, 19, 29,...} }

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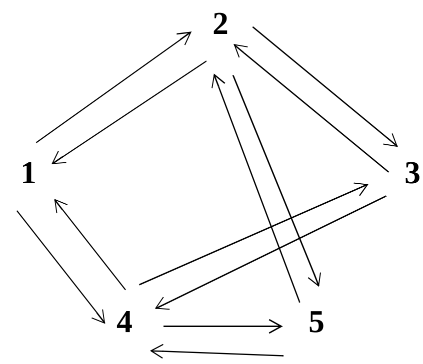
10. (6 points) Graph the relation on {1,2,3,4,5} given as S = {(*a*,*b*) | (*a* + *b*) ≡ 1 MOD 2 }

Since X ≡ 1 MOD 2 means
1 + ? = odd: 2 and 4
2 + ? = odd: 1, 3, and 5
3 + ? = odd: 2 and 4
4 + ? = odd: 1, 3, and 5
5 + ? = odd: 2 and 4

Therefore, S = { (1, 2), (1, 4), (2, 1), (2, 3), (2, 5),
(3, 2), (3, 4), (4,1), (4, 3), (4, 5),
(5, 2), (5, 4)}



11. (6 points) What Equivalence Relation induces the Partition { {1, 2}, {3, 4, 5} } of the set {1, 2, 3, 4, 5}?

Relation = ({1, 2} × {1, 2}) ∪ ({3, 4, 5} × {3, 4, 5})

= { (1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (3, 5), (4, 3), (4, 4), (4, 5), (5, 3), (5, 4), (5, 5) }

12. (10 points) Find the Primary Keys and $P_{3,6}$ for the trumpet database:

Make	**Model**	**Year**	**Serial No.**	**Key**	**Finish**
Conn	38A	1955	105523	Bb	Silver
Conn	40B	1938	52234	Bb	Brass
Conn	22A	1966	212203	C	Silver
Olds	Special	1966	524366	Bb	Silver
Olds	Super	1955	161230	C	Gold
Olds	Mendez	1966	210658	Bb	Silver
Selmer	24A	1953	14522	Bb	Gold
Selmer	Paris	1955	18502	Bb	Silver
Selmer	Radial	1974	64299	C	Brass

Primary Keys = **Model** and **Serial No.**

$P_{3,6}$ = { (1955, Silver), (1938, Brass), (1966, Silver), (1955, Gold), (1953, Gold), (1974, Brass) }

(Note: NO duplicate ordered pairs allowed!)

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13. (6 points) (a) For a collection of 45 coins, if 27 are quarters, 12 are quarters from the 1990's, and 20 are coins from the 1990's. Show that the probability the a coin being a quarter is INDEPENDENT from it being from the 1990's?

Denote E = {Quarters} and F = {Coins from the 1990's}

Test 1: Show P(E | F) = P(E). Since P(E | F) = $|E \cap F| / |F|$ = 12/20 = 3/5 = 27/45 = P(E), we see that E is Independent of F.

Test 2: North = 12, South = (45 - 27 - 20 + 12) = 10, East = (27 - 12) = 15 and West = (20 - 12) = 8, so North(South) = 12(10) = 120 = 15(8) = East(West), so E is Independent of F.

14. (6 points) Find the probability of rolling 2 fair dice and getting a total of at least 10 if the first die rolls at least 4?

	1	***2***	***3***	***4***	***5***	***6***
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

F = (rows 4–6) E = (sums 10, 11, 12)

First Roll	Second Roll(s)
4	6
5	5, 6
6	4, 5, 6

E = {Sum $\geq$ 10} and F = {First = 4, 5, or 6} then $|E \cap F|$ = 6 and |F| = 18, therefore P(E | F) = 6/18 = 1/3.

15. (6 points) (a) Find the Disjunctive Normal Form for the Boolean Polynomial

$$F(w,x,y,z) = w'z + wx'y'$$

$$F(w,x,y,z) = w'z + wx'y' = w'(x + x')(y + y')z + wx'y'(z + z')$$

$$= w'xyz + w'x'yz + w'xy'z + w'x'y'z + wx'y'z + wx'y'z'$$

16. (6 points) Find the Disjunctive Normal Form of the polynomial with Truth Table:

x	*y*	*z*	F(*x,y,z*)		
1	1	1	0		
1	1	0	1	>>>	xyz'
1	0	1	0		
1	0	0	1	>>>	$xy'z'$
0	1	1	0		
0	1	0	0		
0	0	1	1	>>>	$x'y'z$
0	0	0	0		

so $F(x,y,z) = xyz' + xy'z' + x'y'z$