Spring 2011 Examination 1 CMSC 203 Name __________SOLUTION KEY__________

1. (20 points) Circle **T** for True or **F** for False as they apply to the following statements:

T **F** A statement is either a tautology or a contradiction.

T **F** The set {a, e, i, o, u, y} has 64 subsets.

T **F** The empty set is a subset of itself.

T **F** Onto functions map sets to sets of equal or smaller size.

T **F** If $\Sigma = \{0, 1\}$, then $\Sigma^3 = \{000, 111\}$.

T **F** The negation of an implication is an implication.

T **F** The density of a binary string equals its length.

T **F** If $\Sigma = \{0, 1\}$, then for any string s in Σ^8, H(s, 11111111) = d(s).

T **F** The conditional statement and its converse are logically equivalent.

T **F** The set of Rational numbers is countable.

2. (10 points) Use the Laws of Logic to show: $\neg p \wedge (q \vee \neg r) \equiv \neg [(r \to q) \to p]$

$$\neg p \wedge (q \vee \neg r) \equiv \neg p \wedge (\neg r \vee q) \equiv \neg p \wedge (r \to q) \equiv \neg [p \vee \neg (r \to q)]$$

$$\equiv \neg [\neg (r \to q) \vee p]$$

$$\equiv \neg [(r \to q) \to p]$$

3. (6 points) Find the negation of the following Universal Conditional:

Some people who like Math study Economics.

All people like Math and do not study Economics.

4. (10 points) Use the Rules of Inference to show the following is a valid argument:

$$p \to q$$
$$\neg r \to \neg q$$
$$r \to (s \wedge u)$$
$$p$$
$$\therefore s$$

1. $p \to q$ AND p THEREFORE q ;
2. $\neg r \to \neg q$ AND q THEREFORE r ;
3. $r \to (s \wedge u)$ AND r THEREFORE $(s \wedge u)$;
4. $(s \wedge u)$ THEREFORE s.

5. (10 points) Given the alphabet $\Sigma = \{0,1\}$, list the subsets of Σ^4 where each subset contains the elements of Σ^4 that have the same Hamming Distance from 1100.

s	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
H(1100, *s*)	2	3	3	4	1	2	2	3	1	2	2	3	0	1	1	2

H(1100, *s*) = 0: { 1100 }
H(1100, *s*) = 1: { 0100, 1000, 1101, 1110 }
H(1100, *s*) = 2: { 0000, 0101, 0110, 1001, 1010, 1111 }
H(1100, *s*) = 3: { 0001, 0010, 0111, 1011 }
H(1100, *s*) = 4: { 0011 }

6. (10 points) Using the Properties of Sets, to show $(A - B) \cup (A \cap B) \cup (B - A) = A \cup B$.

$$\begin{aligned}(A - B) \cup (A \cap B) \cup (B - A) &= (A \cap B^c) \cup (A \cap B) \cup (B \cap A^c)\\ &= (A \cap B^c) \cup (A \cap B) \cup (A \cap B) \cup (B \cap A^c)\\ &= [\, A \cap (B^c \cup B)] \;\cup\; [(A \cup A^c) \cap B]\\ &= (A \cap U) \cup (U \cap B)\\ &= A \cup B.\end{aligned}$$

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7. (16 points) Given the function F = {(*a*, 2), (*b*, 1), (*c*, 2), (*d*, 1), (*e*, 2)}

(a) What is the Domain of F?

Dom(F) = {*a, b, c, d, e*}

(b) What is the Image of F?

Im(F) = {1, 2}

(c) What is the Inverse of F?

F^{-1} = { (2, *a*), (1, *b*), (2, *c*), (1, *d*), (2, *e*) }

(d) Why or why not is the Inverse in **(c)** a function?

F^{-1} is NOT a function since inputs 1 and 2 each map to multiple outputs.

8. (10 points) Find $F \circ F \circ F$ for F: {0, 1, 2, 3, 4} $\rightarrow$ **Z** given by F(*x*) = 3x – 2.

F = { (0, -2), (1, 1), (2, 4), (3, 7), (4, 10) };

$F \circ F$ ={ (-2, -8), (1, 1), (4, 10), (7, 19), (10, 28) } $\circ$ { (0, -2), (1, 1), (2, 4), (3, 7), (4, 10) }

= { (0, -8), (1, 1), (2, 10), (3, 19), (4, 28) };

$F \circ F \circ F$ = { (-8, -26), (1, 1), (10, 28), (19, 55), (28, 82) } $\circ$ { (0, -8), (1, 1), (2, 10), (3, 19), (4, 28) }

so, $F \circ F \circ F$ = { (0, -26), (1, 1), (2, 28), (3, 55), (4, 82) }

9. (8 points) For the given argument, circle **MP** if it is an example of Modus Ponens, **MT** if it is an example of Modus Tollens, **CE** if it is an example of Converse Error, and **IE** if it is an example of Inverse Error.

MP MT CE IE *All girls like tennis* and *Mary likes tennis*, therefore *Mary is a girl.*

MP MT CE IE *All girl like tennis* and *Mary is a girl*, therefore *Mary likes tennis.*

MP MT CE IE *All girl like tennis* and *Mary is not a girl*, therefore *Mary dislikes tennis.*

MP MT CE IE *All girl like tennis* and *Mary dislikes tennis*, therefore *Mary is not a girl.*