CMSC203 - Discrete Structures - Final Examination - Spring 1999

Part One

1. Using the Properties of Sets, show that if A and B are sets, then $A \cap (A \cup B) = A$.
2. Find the Power Set of $\{1,\{1\}\}$.
3. Show that if p,q, and r are statements, then $p \to (q \wedge r) \equiv (p \to q) \wedge (p \to r)$.
4. Show the following is a valid argument:

$$\begin{array}{l} p \to q \\ r \vee s \\ r \to t \\ \sim q \\ s \to p \\ \therefore\ t \end{array}$$

5. Find the Disjunctive Normal Form of the Boolean Polynomial $F(x,y,z) = xy' + z$
6. What is the negation of the statement: *All integers that are even are divisible by 2.*
7. What is universal modus tollens? Draw a picture to illustrate it.
8. Prove one of the following statements:

a. The square root of an irrational number is irrational. b. If n is an odd integer, then $8 \mid (n^2 - 1)$.

9. Use the Division Algorithm to find 37 MOD 5 and 37 DIV 5.
10. Use the Euclidean Algorithm to find GCD(274,136).

Part Two

11. Rewrite $\sum_{i=1}^{10} i^{10-i}$ as a summation from 0 to 9.
12. Prove one of the two statements by Mathematical Induction:

a. If n is a positive integer, $\sum_{i=0}^{n} 7^i = \frac{7^{n+1} - 1}{6}$. b. Every positive integer has a binary representation.

13. Draw the directed graph of the relation $R = \{(a,b) \mid a,b \in \{1,2,3,4,5,6\}$ and $a + b$ is even$\}$
14. Show the relation Congruence Modulo 7 is an Equivalence Relation.
15. If $f:\{1,2,3,4\} \to \{5,7,9,11\}$ and $g:\{1,3,5,7,9,11\} \to \{2,10,26,50,82,122\}$ are given by $f(x) = 2x + 3$ and $g(x) = x^2 + 1$, calculate $(g \circ f)^{-1}$
16. Let $f:\mathbf{Z} \to \mathbf{Z}_{\text{even}}$ be given by $f(x) = 2x + 4$. Show f is a bijection.

Part Three

17. How many 8-character license plates made up of 26 letters and 10 digits begin with "A1" and end with "Z9"?
18. How many ways can 2 pennies, 3 nickels, 4 dimes, and 5 quarters be ordered in a line?
19. How many integer solutions are there to the equation: $a + b + c + d + e + f = 50$ with $a \geq 2$, $b \geq 4$, $c \geq 6$, $d \geq 1$, $e \geq 3$, and $f \geq 5$?
20. TRUE or FALSE? C(7,0) + C(7,1) + ... + C(7,7) = 128.
21. Use the Iteration Method to solve $s_n = 7s_{n-1} + 1$, when $s_0 = 1$.
22. Find s_{152637} given $s_n = 3s_{n-1} + 28s_{n-2}$ and $s_0 = 2$ and $s_1 = 5$.