Examination 1 - Spring 1999

Symbols: **N** denotes the Natural Numbers, **Z** denotes the Integers, **Q** denotes the Rational Numbers, **I** denotes the Irrational Numbers, and **R** denotes the Real Numbers, $\mathscr{P}$(A) is the Power Set of a set A.

1. Circle T if the statement is true or F if the statement is false.

T F **R** – **I** = **Z**.

T F $\varnothing \in \mathscr{P}(\{1, 2, 3\})$. .

T F The contrapositive of the statement, ***I go to class implies there was no snowstorm*** is the statement, ***There was no snowstorm implies I did not go to class.***.

T F The following is a valid argument:

$$\begin{array}{l} \sim p \wedge q \\ q \rightarrow (r \vee s) \\ r \rightarrow p \\ \therefore\ s \end{array}$$

T F If A = {00,11}, then A × A = {0000, 0011, 1100, 1111}.

T F If $\Sigma = \{goo, ga\}$ is an alphabet, then *googoogaga* $\in \Sigma^4$.

T F The set of prime numbers and the set of composite numbers partition the set of integers.

T F If A, B, and C are sets, then $(A \cap B \cap C)^c = A^c \cap B^c \cap C^c$.

T F If *a*,*b*, and *c* are integers with $a = bc$, then a MOD b = a MOD c = 0.

T F The Disjunctive Normal Form of the Boolean Polynomial $F(x, y, z) = xyz$ is $F(x, y, z) = xyz$.

2. Use the Euclidean Algorithm to find gcd(340, 220).

3. Given the informal language statement: ***Every prime number greater than 2 is odd***

a. Rewrite the statement in as a Universal Conditional.

b. Find the negation of the statement (in either formal or informal language).

4. Show the Absorption Law, **without** using truth tables, that $p \vee (p \wedge q) \equiv p$.

5. Given a circuit of three switches controlling a light bulb in such a way that if the first switch is the opposite of the second and third, then the bulb turns on, find:

a. the truth table of the circuit.**b.** the Boolean polynomial of the circuit, in Disjunctive Normal Form.

6. For the sets A = {*w*,*x*,*y*,*z*}, B = {*w*,*z*} and C = {0,1}, verify that (A – B) × C = (A × C) – (B × C)

7. Prove 2 of the 4 theorems below, using the indicated method:

Theorem 1: For all integers, *n*, if n^3 is even, then *n* is even. (By the Method of Contraposition)

Theorem 2: If *a*, *b* and *c* are positive integers such that $a = b + c$, then gcd(*a*,*b*) = gcd(*b*,*c*).

Theorem 3: If A, B, and C are sets, then $C - (A \cap B) = (C - A) \cup (C - B)$.

Theorem 4: The difference of the squares of an integer and its predecessor is odd.