Discrete Structures - Spring 1995 - Exam 2

Symbols: $\mathbb{N}$ denotes the Natural Numbers, $\mathbb{Z}$ denotes the Integers,
$\mathbb{Q}$ denotes the Rational Numbers, and $\mathbb{R}$ denotes the Real Numbers.

1. Circle T if the statement is true or F if the statement is false.

T F If A is a set, then AxA is a function.

T F If A is a set, then AxA is an equivalence relation.

T F If the sets A, B, C, and D partition a set X, then the relation R = (AxA) U (BxB) U (CxC) U (DxD) is an equivalence relation on X.

T F If **f** is a 1-1 and ONTO function from A to B, then A = B.

T F If **f** is a 1-1 function from A to B, then |A| = |**f**(A)|.

T F If $\mathbf{f}$:A $\rightarrow$ B and $\mathbf{g}$:B $\rightarrow$ C are functions, then $(\mathbf{g} \circ \mathbf{f})^{-1} = (\mathbf{g}^{-1} \circ \mathbf{f}^{-1})$.

T F If n is a positive integer, $1 + 10 + 100 + \ldots + 10^{(n-1)} = (10^n - 1)/9$.

T F Let DIV denote the INTEGER DIVIDE operation. If a, b, and p are positive integers and $a \equiv b$ mop p, then [a – p(a DIV p)] = [b – p(b DIV p)].

T F If $\mathbf{H}$ is the Hamming distance function, $\mathbf{d}$ is the density function, and $\mathbf{0}$ is the all-zero string, then $\mathbf{H}(s,\mathbf{0}) = \mathbf{d}(s)$ for all binary strings, s.

T F There cannot exist a 1-1 correspondence between $\mathbb{N}$ and $\mathbb{Q}$.

2. Rewrite: $\frac{1}{1 \cdot 2} + \frac{2}{2 \cdot 3} + \frac{4}{3 \cdot 4} + \frac{8}{4 \cdot 5} + \frac{16}{5 \cdot 6} + \ldots + \frac{512}{10 \cdot 11}$ into summation notation.

3. Let R = {(a,b) | a,b $\in$ {1,2,3,4} and a/b $\leq$ 1}. Graph R.

4. Do 1 of the following 2 induction proofs:

Prove: $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$.

or

Prove: If s_i is a recursively generated sequence given by $s_i = s_{i-1} + s_{i-2}$, with $s_0 = 5$ and $s_1 = 15$, then s_i is divisible by 5 for all $i > 1$.

5. List out the ordered pairs that make up the equivalence relation is induced by the partition {2}, {1,3}, {0,4,5} of the set {0,1,2,3,4,5}?

6. Find the Domain and Image of the function **f** = {(a,3),(w,12),(e,2),(s,5),(t,3),(g,2),(v,1)}.

7. Let **d** be the density function on Σ^n, the set of all n-long strings and define the relation R on Σ^n

R = {(s,t) | s,t $\in \Sigma^n$ and $\mathbf{d}(s) = \mathbf{d}(t)$}.

(a) Show R is an equivalence relation on Σ^n. (b) Describe the partition of Σ^n induced by R.

8. Prove the function $\mathbf{f}$:$\mathbb{R} \rightarrow \mathbb{R}$ given by **f**(x) = 5x + 3 is a bijection (1-1 and onto).