Exam 2 - Discrete Structures - Spring 2008

1. Circle **T** if the corresponding statement is True or **F** if it is False.

T **F** The sequence {1, –1, 1, –1, 1, –1 ...} is an example of an Alternating sequence.

T **F** The Weak and Strong Principles of Mathematical Induction are logically equivalent.

T **F** In general, iterative algorithms use memory more efficiently than their equivalent recursive representations.

T **F** $1 + 10 + 100 + 1000 + ... + 10^n = 10^{(n+1)} - 1$.

T **F** 2 + 4 + 6 + 8 + 10 + ... + 2,000 = 1,001,000.

T **F** Algorithms whose order is $O(n^2)$ are less efficient than those of order $O(2^n)$.

T **F** If the set A contains the element x, then half the subsets of A contain x.

T **F** $\text{LCM}(p,q)\text{GCD}(p,q) = pq$.

2. Let $\{a_n\}$ and $\{b_n\}$ be the sequences defined, for $n \geq 0$, by:

$a_n = (n - 1)$, and $b_n = (n + 1)$. Find c_1, c_2, c_3, and c_4 when $c_n = (a_n)(b_n)$.

3. Write out the Euclidean Algorithm and trace its steps to calculate GCD(21,4).

4. **(a)** Give a Recursive Definition for the odd integers.

(b) Find the Big-Oh of the algorithm with complexity: $(n^2 + 1)(4n^3 + 5) + [2n^4 + n(\log n)](3n^2)$.

5. (a) Given $a = 2^2 3^3 5^5 7^7 11^9 13^{11} 17^{13} 19^{15} 23^{17} 29^{19} 31^{21}$. and $b = 2^1 3^2 5^4 7^8 11^{16} 13^{32} 17^{64} 19^{128} 23^{256} 29^{512} 31^{1024}$,

GCD(a,b) = _____ LCM(a,b) = _____.

(b) List out the search intervals in applying the Binary Search algorithm to find 7 in the list:

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

(Hint: Include the mid/test point with the lower interval.)

6. Prove one of the two Theorems below using Mathematical Induction.

<u>Theorem 1</u>: For all integers $n \geq 0$, $\displaystyle\sum_{i=0}^{n} 3^i = \frac{3^{n+1} - 1}{2}$.

<u>Theorem 2</u>: If $a_0 = 0$, $a_1 = 10$, and $a_2 = 20$, then $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ is divisible by 10, for all $n \geq 3$.

7. Prove one of the two Theorems below:

<u>Theorem 1:</u> If a, b, p, and r are Integers with $a = bq + r$, then GCD(a,b) = GCD(b,r).

<u>Theorem 2:</u> The product of odd integers is odd.

8. Prove one of the two Theorems below by Contradiction or Contraposition.

<u>Theorem 1:</u> $\sqrt{2}$ is irrational.

<u>Theorem 2:</u> The Prime numbers form an infinite set.