CMSC 203 - Exam 1 - Spring 2002

Notation: Let $\mathbf{R}$ denote the Real Numbers, and P(A) denote the Power Set of A.

1. (20 pts.) Circle T if the statement is true or F if the statement is false.

T F $\mathbf{Z} \times \mathbf{Z} \subset \mathbf{R} \times \mathbf{R}$.

T F If n is an Natural Number, the set $\{1,2,3,...,n\}$ has n^2 subsets.

T F For any set A, $\varnothing \subset$ P(A) and $\varnothing \in$ P(A).

T F The negation of the statement: **All Natural Numbers are even** is the statement: **Some Natural Numbers are not even**.

T F [(36 DIV 5) – (93 MOD 7)] = 5.

T F If $d \mid (x + y)$, then $d \mid x$ and $d \mid y$.

T F If A = {0,1}, then A × A × A = {000,001,010,011,100,101,110,111}.

T F If $\Sigma = \{0,1\}$, then $\Sigma^5 = \Sigma \times \Sigma \times \Sigma \times \Sigma \times \Sigma$.

T F The set of even integers and the set of odd integers partition the set of integers.

T F A conditional statement and its contrapositive are logically equivalent.

2. (6 pts.) Use the Euclidian Algorithm to find gcd(1000,60)

3. (10 pts.) Show, without using truth tables, that $(\sim p \wedge q) \rightarrow r \equiv \sim p \rightarrow (q \rightarrow r)$.

4. (4 pts.) Give the converse, inverse, contrapositive, and negation of the universal statement: All prime numbers greater than 2 are odd.

5. (10 pts.) Find the Disjunctive Normal Form of a circuit of four inputs in such a way that if the integer value of the inputs is prime, then current flows out of the circuit. (For example, 12 is not prime, and 12 = 1100, so f(1100) = 0)

6. (10 pts.) Show the following is a valid argument:

$$\begin{array}{l} p \rightarrow (q \wedge r) \\ \underline{\sim r \qquad\qquad} \\ \therefore \sim p \end{array}$$

7. (40 pts.) Prove 2 of the 4 theorems:

Theorem 1: $(A \cup B) \cap (A \cup C^c) \cap (B^c \cup C^c) = (A - B) \cup (B - C)$

Theorem 2: For all integers a and b, if b is the successor of a, then $b^2 - a^2$ is odd.

Theorem 3: If every integer greater than 1 can be factored as the product of primes,then there is no largest prime.

Theorem 4: If a, b, and c are integers with $a = b + c$, then $\gcd(a,b) = \gcd(b,c)$.