Exam 2 - CMSC203 - Discrete Structures - Spring 2001

1. Circle **T** of the corresponding statement is True and **F** if it is False:

T **F** $1 + 2 + 3 + 4 + ... + 1{,}000 = 1{,}010$

T **F** If A is a non-empty set, then A x A is an Equivalence Relation.

T **F** If A is a non-empty set, then A x A is function.

T **F** If f:X $\rightarrow$ Y is a function and f(X) = Y, then f is a ONE-TO-ONE function.

T **F** If A is a non-empty set, then $\varnothing$ is the *smallest* Equivalence Relation on A.

T **F** |**N**| = |**Q**|.

T **F** If f and g are functions with f:X $\rightarrow$ Y and g:Y $\rightarrow$ Z, then $f \circ g = g \circ f$.

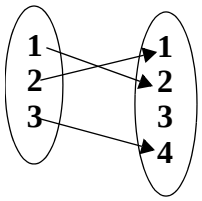
T **F** If f:A $\rightarrow$ B is a function and i_B:B $\rightarrow$ B is the identity function on B, then $(i_B \circ f) = f$.

T **F** The Weak Form of Mathematical Induction and the Strong Form are equivalent.

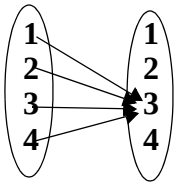
T **F** If R is an Equivalence Relation on a set A, and $a,b \in$ A with [a] = [b], then $(a,b) \in$ R.

2. Circle F for function, I for one-to-one, S for onto, and B for one-to-one correspondence as the properties apply to the relations below. More than one choice or no choices may apply to a given relation.

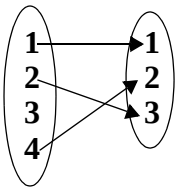
a. F I S B



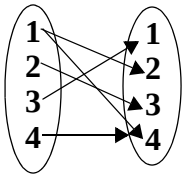
b. F I S B



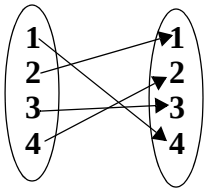
c. F I S B



d. F I S B



e. F I S B



3. Write $\frac{a^2}{1^3} + \frac{a^3}{2^4} + \frac{a^4}{3^5} + \ldots + \frac{a^{19}}{18^{20}}$ in summation of **i** notation (starting with i = 1), then rewrite as a summation ranging from **j** = 9 to 26.

4. Let $\Sigma = \{0,1\}$and let H be the Hamming distance function on binary strings. Consider the relation:

$$R = \{(s,t) \mid s,t \in \Sigma^4 \text{ and } H(s,0000) = H(t,0000)\}.$$

a. Prove that R is an Equivalence Relation. **b.** What partition of Σ^4 does the relation R induce?

5. Let f:**R** $\rightarrow$ **R** be the function $f(x) = 9x + 5$. **a.** Show that that f is 1-1 and onto. **b.** Find $f^{-1}(x)$.

6. Let $f = \{(1,9),(2,7),(3,5),(4,3),(5,1)\}$ and let $g = \{(1,8),(3,6),(5,4),(7,2),(9,0)\}$.
Show $(g \circ f)^{-1} = (f^{-1} \circ g^{-1})$.

7. Prove 1 of the following 2 statements using the indicated method:

a. Using Strong Induction, show that if n is an integer greater than 1, then n has a prime factor.

b. Using Weak Induction, show that $1 + 2 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 1$.