Exam 1 - Discrete Structures - CMSC203 - Spring 2001

Symbols: **N** denotes the Natural Numbers, **Z** denotes the Integers, **Q** denotes the Rational Numbers, **I** denotes the Irrational Numbers, and **R** denotes the Real Numbers, $\mathscr{P}$(A) is the Power Set of a set A.

1. Circle T if the statement is true or F if the statement is false.

T F $\mathbf{I} - \mathbf{Q} = \varnothing$.

T F If U is a universal set, then U $\in \mathscr{P}$(U).

T F The negation of the statement:***No integers are irrational*** is the statement: ***No integers are rational***.

T F The following is a valid argument:

$$\begin{array}{l} \sim p \rightarrow q \\ \sim r \vee q \\ \underline{p \wedge r} \\ \therefore r \end{array}$$

T F If A = {1,11} and B = {111,1111}, then A × B = {(1, 111), (1, 1111), (11, 111), (11, 1111)}.

T F If Σ is an alphabet, then $\Sigma^3 \subseteq \Sigma^5$.

T F According to Universal Modus Tollens, the argument: **All that glitters is gold** AND **my watch is not gold** THEREFORE **my watch does not glitter** is valid.

T F If A and B are sets, then A $\cap$ (B $\cup$ A) = A .

T F If a,b and c are integers with $a = b + c$, then GCD(a,b) > GCD(b,c).

T F For any Boolean Polynomial, F(x_1,x_2,...,x_n), **CNF**(F) = [**DNF**(F)]'.

2. Using the Laws of Logic, show that $\sim[(p \wedge q) \vee (r \wedge q)] \equiv (p \vee r) \rightarrow \sim q$.

3. Draw a diagram to illustrate the valid argument:

For all integers, x > 2, if x is prime, then x is odd.
36 is even.
Therefore, 36 is not prime.

4. Expressing 220 in the form of the Quotient-Remainder Theorem using the modulus 6, find:

a. 220 *div* 6 = _______ **b.** 220 *mod* 6 = _______ **c.** Find GCD(220,6) using the Euclidean Algorithm

5. Find the Disjunctive Normal Form of the Boolean Polynomial F(x,y,z) = y.

6. For the sets A = {a,b,c}, B = {d,e,f} and Y = {x,y}, verify that (A − B) × Y = (A × Y) − (B × Y)

7. Prove 2 of the 4 theorems below, using the indicated method:

Theorem 1: The set of prime numbers is infinite. (By the Method of Contradiction)

Theorem 2: The odd integers are CLOSED under multiplication.

Theorem 3: If A, B, and C are sets, then A − (B $\cup$ C) = (A − B) $\cap$ (A − C). (By the Properties of Sets)

Theorem 4: The successor of the product of successive even integers is perfect square.