Sample Exam 2 - Fall 1999 - CMSC 203 / Discrete Structures

Symbols: **N** denotes the Natural Numbers, **Z** denotes the Integers, **Q** denotes the Rational Numbers, and **R** denotes the Real Numbers.

1. Circle T if the statement is true or F if the statement is false.

T F If A is a non-empty set, then $\varnothing$ is a the smallest equivalence relation on A.

T F If A is a non-empty set, then A×A is the largest equivalence relation on A.

T F Let R = (A×A) ∪ (B×B) ∪ (C×C) ∪ (D×D) is an equivalence relation on a set X. Then the sets, A, B, C, and D, partition X .

T F One-to-one functions map larger finite sets into smaller finite sets.

T F If a function is onto, its range (co-domain) equals its image.

T F If f:A → B and g:B → A are 1-1 and onto functions, then $g \circ f = f \circ g$.

T F If n is a positive integer, $9(1 + 10 + 10^2 + ... + 10^{(n-1)}) = (10^n - 9)$.

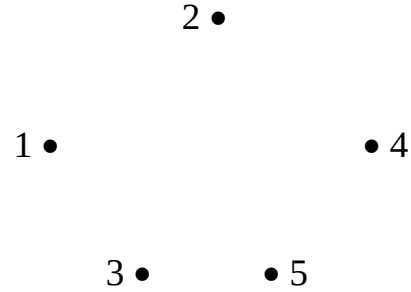
T F The Weak and Strong Forms of Mathematical Induction are equivalent.

T F If H is the Hamming distance function, then H(111000 , 000000) = H(111000 , 111111).

T F There are as many prime numbers as rational numbers.

2. Rewrite: $\frac{1}{1 \cdot 2} + \frac{2}{2 \cdot 3} + \frac{4}{3 \cdot 4} + \frac{8}{4 \cdot 5} + \frac{16}{5 \cdot 6} + \ldots + \frac{512}{10 \cdot 11}$ as a sum from 6 to 15.

3. Let R = $\{(a,b) \mid a,b \in \{1,2,3,4,5\}$ and $a + b \leq 5\}$. Graph R.



4. Prove 1 of the 2 Theorems below by Induction:

Theorem: A set with n elements has 2^n subsets.

Theorem: Every integer greater than 1 can be factored as the product of prime numbers.

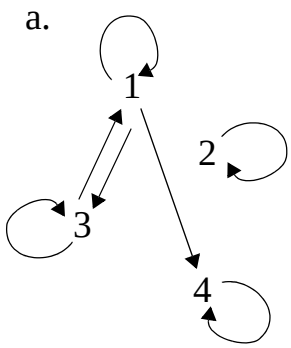
5. Let f = {(1,3),(2,1),(3,5),(4,4),(5,2)} and g = {(1,2),(2,3),(3,4),(4,5),(5,1)}.

Show that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

6. For the relations graphed below, circle:

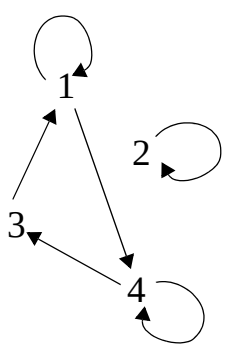
R if it is REFLEXIVE
S if it is SYMMETRIC
A if it is ANTI-SYMMETRIC
T if it is TRANSITIVE
N if it is NONE of these.

a.



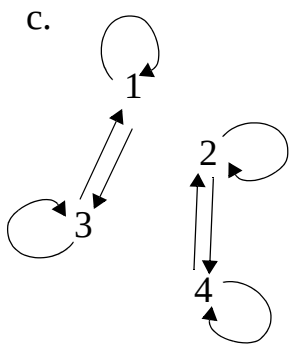
R S A T N

b.



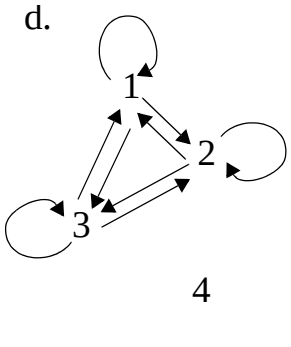
R S A T N

c.



R S A T N

d.



R S A T N

7. Let R be the relation on **Z** given by R = $\{(a,b) \mid a,b \in \mathbf{Z} \text{ and } a^2 = b^2\}$.
(a) Show R is an equivalence relation on **Z**.
(b) Describe the partition of **Z** induced by R.

8. Prove the function $f{:}\mathbf{R} \to \mathbf{R}$ given by $f(x) = \dfrac{x+5}{3}$ is a bijection (one-to-one and onto).