Sample Exam 1 - Fall 1999 - CMSC 203 / Discrete Structures

Symbols: **N** denotes the Natural Numbers, **Z** denotes the Integers, **Q** denotes the Rational Numbers, **I** denotes the Irrational Numbers, and **R** denotes the Real Numbers, $\mathscr{P}$(A) is the Power Set of a set A.

1. Circle T if the statement is true or F if the statement is false.

T F $\mathbf{I} \cap \mathbf{Z} = \{0\}$

T F $\{1, \varnothing\} \in \mathscr{P}(\{1, \varnothing\})$.

T F The inverse of the statement, ***If you mow my lawn, then I will pay you $25*** is the statement, ***I do not pay you $25 implies you do not mow my lawn***.

T F The following is a valid argument:

$$\begin{array}{l} \sim t \wedge r \\ \sim t \rightarrow (q \vee p) \\ p \rightarrow \sim r \\ \underline{q \rightarrow s} \\ \therefore s \end{array}$$

T F If A = {*xy*, *yx*}, then A × A = {*xyxy*, *xyyx*, *yxxy*, *yxyx*}.

T F If $\Sigma = \{xy, yx\}$ is an alphabet, then $xyxyyxxyyx \in \Sigma^5$.

T F There is no prime number that divides both a positive integer and its successor.

T F If A and B are sets, then $A \cup (A \cap B) = A$.

T F If a,b,q, and r are integers with $0 \leq r < b$ and $a = bq + r$, then $\gcd(a,b) = \gcd(q,r)$.

T F The Disjunctive Normal Form of the Boolean Polynomial $F(x, y) = 1$ is $F(x, y) = xy + xy' + x'y + x'y'$.

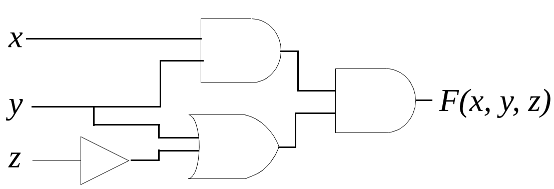
2. Use the Euclidean Algorithm to find gcd(840,144).

3. a. Negate the statement: ***The square root of any positive integer is positive.***

b. If you apply the *Converse Error* to the argument: ***Every integer is rational*** AND ***q is rational***, what conclusion follows?

4. Show, **without** using truth tables, that $p \rightarrow \sim(q \wedge \sim r) \equiv (p \wedge q) \rightarrow r$.

5. Find the Truth Table and Disjunctive Normal Form of the following circuit:



6. For the sets A = {*a*,*b*,*c*}, B = {*a*,*d*,*e*} and Y = {0,1}, verify that $(A - B) \times Y = (A \times Y) - (B \times Y)$

7. Prove 2 of the 4 theorems below, using the indicated method:

Theorem 1: For all integers, n, if n^3 is odd, then n is odd. (By Contraposition)

Theorem 2: The only positive integer that divides any integer and its successor is 1. (By Contradiction)

Theorem 3: If a and b are distinct integers then there is a rational number between them.

Theorem 4: The difference of the squares of *successive* integers is an odd integer.