CMSC 203 - Discrete Structures - Fall 1997 - Final Examination

1. Circle **T** or **F** as it applies to the associated statement below:

T F The negation of the statement, "Some integers are positive," is "Some integers are non-positive."

T F The following is a valid argument:

$$\begin{array}{l} \sim q \wedge p \\ \sim q \rightarrow (t \vee s) \\ s \rightarrow \sim p \\ \underline{t \rightarrow r} \\ \therefore r \end{array}$$

T F If $p \equiv q$, then $p \leftrightarrow q$ is a tautology.

T F If A = {1,3}, B = {1,2,5}, and U = {0, 1, 2, 3, 4, 5}, then $(B \cup A^c)^c = \{3\}$

T F If A = {x, y} and B = {a, b}, then B × A = {(a, x), (a, y), (b, x), (b, y)}

T F If f :{a, b, c, d, e} → {v, w, x, y, z} is defined as f = {(a,x), (b,z), (c,x), (d,w), (e,y)}, then f is an ONTO function.

T F If f :{a, b, c, d, e} → {v, w, x, y, z} is defined as f = {(a,x), (b,z), (c,x), (d,w), (e,y)}, then f is an ONE-TO-ONE function.

T F If the relation **R** on A = {0, 1, 2, 3, 4} is **R** = {(a, b) | a, $b \in$ A and $b \equiv 4a$ mod 5}, then **R** = {(0, 0), (1, 4), (2, 3), (3, 2), (4, 1)}

T F The relation {(1, 1), (2, 2), (3, 3), (4, 4)} is both SYMMETRIC and ANTI-SYMMETRIC on the set {1, 2, 3, 4}.

T F Let S = {0,1}, H(s, t) be the Hamming Distance Function, and define the equivalence relation R = {(s, t) | $s, t \in \Sigma^4$ and H(s, 0000) = H(t, 0000)}. Then [0011] = {0011, 0000}.

T F There are $\frac{14!}{4! \cdot 6! \cdot 4!}$ distinct orderings of the letters *abbabccacbcabb*.

T F If A, B, and C are sets which partition a set X, then |A| = |X| – |B| – |C|.

T F If n and r are positive integers with $n \geq r$, then P(n, r) = nC(n, r).

T F The Characteristic Polynomial of $s_n = s_{n-3} + s_{n-5}$ is $x^5 - x^3 - 1$.

T F If a recurrence relation has the General Solution: $s_n = (A + Bn + Cn^2)(3^n)$, then its Characteristic Polynomial is $(x - 3)^3$.

2. Fill in the blanks so the function g:{a, b, c, d}→{w, x, y, z} is a 1-1 correspondence.

g = {(a, ____), (b, ____), (c, ____), (d, ____)}.

3. Find the Boolean Polynomial for a circuit of 5 inputs which outputs a current whenever the first three inputs are the opposite of the last two.

4. How many distinct license plates are there consisting of either 8 non-repeated digits or 3 non-repeated capital letters followed by 5 non-repeated digits?

5. How many different ways can Andrew, Betty, Charles, Diane, Edward, Fay, Gordon, Harriet, Isaac, and June sit around a circular table so that Andrew and Betty never sit next to one another?

6. Show that $\binom{n}{n-2} = \frac{n(n-1)}{2}$.

7. How many integer solutions are there to the equation $a + b + c + d + e + f + g = 50$ provided $a \geq 1$, $b \geq 2$, $c \geq 3$, $d \geq 4$, $e \geq 5$, $f \geq 6$, and $g \geq 7$?

8. Given the recurrence relation $s_n = 4s_{n-1} + 21s_{n-2}$, what is s_{999} when $s_0 = 7$ and $s_1 = -1$?

9. Prove ONE of the TWO statements below:

a. If d, n, q, and r are integers with $n = dq + r$, then GCD(n, d) = GCD(d, r).

b. The square root of 2 is an irrational number.

10. Prove ONE of the TWO statements below:

a. $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

b. If $a_1, a_2, a_3, \ldots$ is the sequence: $a_0 = 3$, $a_1 = 5$, $a_2 = 7$ with $a_n = a_{n-1} + a_{n-2} + a_{n-3}$, then a_n is odd for all $n \geq 3$.

11. Prove ONE of the TWO statements below:

a. The function f: **R** $\rightarrow$ **R** given by $3y + 2x = 6$ is a bijection.

b. The relation R on **Z** given by R = $\{(a, b) \mid a, b \in \mathbf{Z}$ and $b \equiv a \bmod 5\}$ is an equivalence relation.