Discrete Structures - Examination 2 - Fall 1997

Name ______________________ Show All Work!

1. Circle **T** of the corresponding statement is True and **F** if it is False:

T **F** 1 + 2 + 3 + 4 + ... + 1,000 = 1,001,000

T **F** If A is a non-empty set, then A x A is an Equivalence Relation.

T **F** If f is a function whose image and range are the same set, then f is ONTO.

T **F** If g:A $\rightarrow$ B is a function and $|g(A)| = |B|$, then g is a ONE-TO-ONE function.

T **F** If A = $\{a,b,c\}$, then the relation R = $\{(a,a)\}$ is the *smallest* Equivalence Relation on A.

T **F** $|\mathbf{N}| = |\mathbf{Q}|$.

T **F** If H is the Hamming distance function on binary strings, then H(10110011,11110000) = 4.

T **F** If f:A $\rightarrow$ B is a function and i_B:B $\rightarrow$ B is the identity function on B, then $(i_B \circ f) = f$.

T **F** If two binary strings have the same density, then they are equal.

T **F** If R is an Equivalence Relation on a set A, and $a,b \in$ A with [a] = [b], then $(a,b) \in$ R.

2. Describe the Hamming distance function as the composition of two functions.

3. Write $\frac{a^2}{1^3} + \frac{a^3}{2^4} + \frac{a^4}{3^5} + \ldots + \frac{a^{19}}{18^{20}}$ as a summation ranging from i = 5 to 22.

4. Let $\Sigma = \{0,1\}$, let d(.) be the density function on binary strings. and

let R = $\{(s,t) \mid s,t \in \Sigma^4$ and $d(s) = d(t)\}$.

a. Prove that R is an Equivalence Relation.

b. What partition of Σ^4 does the relation R induce?

5. Let f:**R** $\rightarrow$ **R** be the function $f(x) = 5x + 9$.

a. Prove that f is 1-1 and onto.

b. Find $f^{-1}(x)$.

6. Let $f = \{(1,9),(2,7),(3,5),(4,3),(5,1)\}$ and let $g = \{(1,8),(3,6),(5,4),(7,2),(9,0)\}$. Find $f^{-1} \circ g^{-1}$.

7. Prove 1 of the following 2 statements using the indicated method:

a. Using Strong Induction, show that if n is an integer greater than 1, then n has a prime factor.

b. Using Weak Induction, show that an n-element set has 2^n subsets.