Discrete Structures - Fall 1996 - Examination 1

1. (20 pts.) Circle **T** if the statement is true or **F** if the statement is false.

T **F** $\mathbf{N} \cup \mathbf{Z} = \mathbf{Q}$.

T **F** $\{1,2,3\} \in \mathscr{P}(\{1,2,3,4\})$. (**Note:** $\mathscr{P}(A)$ denotes the Power Set of A.)

T **F** The **negation** of the statement ***All students are motivated and not lazy*** is the statement ***Some students are either not motivated, or are lazy, or both.***

T **F** The **inverse** of the statement ***If I pass CMSC 203, then I get a good job*** is: ***I pass CMSC 203, but I do not get a good job.***

T **F** If $A = \{x,y,z\}$, then $A \times A = \{(x,x),(x,y),(x,z),(y,x),(y,y),(y,z),(z,x),(z,y),(z,z)\}$.

T **F** If $\Sigma = \{x,y,z\}$, then $xyz \in \Sigma^2$.

T **F** The sets $\{1,3,5\}$, $\{1,2,3\}$, and $\{3,4,5\}$ partition the set $\{1,2,3,4,5\}$.

T **F** If d divides n, then $n/d = n \bmod d$.

T **F** For all integers $x > 1$, if x is prime, then x is odd.

T **F** A set with 8 elements has 255 non-empty subsets.

2. (6 pts.) Use the Euclidean Algorithm to find gcd(744,122).

3. (10 pts.) Find the truth table of the Boolean Polynomial $\mathbf{F(x,y,z) = xy' + (x' + z)'}$.

4. (7 pts.) Show the following is a valid argument:

*If **x** is not rational, then **xy** is real*
***z** is an even integer and **xy** is real*
*__**x** is rational or **z** is a prime integer__*
*$\therefore$ **z** is a prime integer.*

5. (10 pts.) Given the quantified statement: *For all $x \in \mathbf{Z}$, if x is odd, then $(x + 1)$ is even* , find its:

Converse:_____

Inverse:_____

Contrapositive:_____

Negation:_____

6. (7 pts.) For the sets $A = \{1,2,3,4,5\}$, $B = \{2,4,6\}$, and $C = \{1,2,4,8\}$ from the Universal Set $U = \{0,1,2,3,4,5,6,7,8,9\}$, show that: $(A \cap B^c)^c - (A \cap B) = A^c$.

7. (40 pts.) Prove 2 of the 4 theorems:

Theorem 1: For all integers n, if n^3 is even, then n is even.

Theorem 2: $\sqrt[3]{2}$ is irrational. (Hint: Assume Theorem 1 is true.)

Theorem 3: If n and d are positive integers and (n div d) = (n mod d), then $(d + 1) \mid n$.

Theorem 4: For all non-zero integers a and b, if $a \mid b$ and $b \mid a$, then $a = b$.