

Discrete Structures - Fall 1995 - Exam 2

1. Let S be the relation defined on \mathbf{Z} as $S = \{(x,y) \mid x,y \in \mathbf{Z} \text{ and } x^2 = y^2\}$.

(a) Prove S is an equivalence relation.

(b) Describe the partition of \mathbf{Z} induced by S .

2. Draw the directed graph of the relation R on $A = \{1,2,3,4,5,6,7,8\}$ defined as

$$R = \{(a,b) \mid a,b \in A \text{ and } a \equiv b \pmod{5}\}.$$

3. Let $f = \{(1,d),(2,c),(3,a),(4,b),(5,e)\}$ and $g = \{(a,4),(b,5),(c,1),(d,2),(e,3)\}$ be functions. Find: (a) $f \circ g$ (b) $g \circ f$ (c) $(g \circ f)^{-1}$

4. For the mappings drawn below, circle the BEST CHOICE from the list of properties:

N if the mapping is NOT a function;

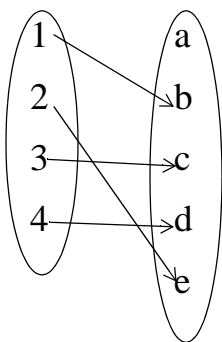
F if the mapping is a function;

I if the mapping is an INJECTIVE (1-1) function;

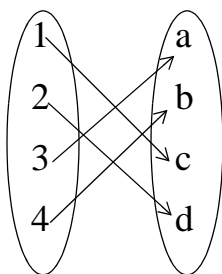
S if the mapping is a SURJECTIVE (onto) function;

B if the mapping is a BIJECTIVE (1-1 and onto) function;

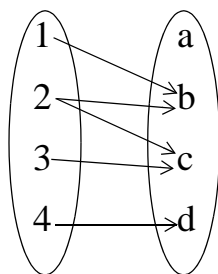
N F I S B



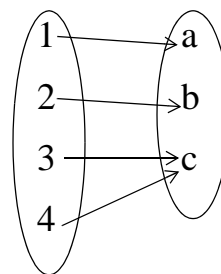
N F I S B



N F I S B



N F I S B



5. Which of the strings 01010101, 11110000, or 00001111 has the smallest Hamming distance from the string 10110011?

6. Given $\Sigma = \{0,1\}$ and the Equivalence Relation R on Σ^8 defined as

$R = \{(s,t) \mid s,t \in \Sigma^8 \text{ and the middle 6 bits of } s = \text{middle 6 bits of } t\}$,
what is $[00000000]$?

7. Rewrite: $\frac{1}{1 \cdot 2} + \frac{2}{2 \cdot 3} + \frac{4}{3 \cdot 4} + \frac{8}{4 \cdot 5} + \frac{16}{5 \cdot 6} + \dots + \frac{512}{10 \cdot 11}$ using the summation:

$$\sum_{k=4}^{13} \underline{\hspace{2cm}}$$

$k = 4$

8. Prove, by *Mathematical Induction*, that $\sum_{i=2}^n i(i-1) = \frac{n(n-1)(n+1)}{3}$