Exam 1 - Fall 1995

1. (20 pts.) Circle T if the statement is true or F if the statement is false.

T F $N - Q = \varnothing$.

T F The set {0,1,2,3,4} has 16 subsets.

T F The Power Set of $\{\varnothing,0,1\}$ contains $\{\varnothing\}$.

T F The negation of the statement: **For all integers, x, if x^2 is even, then x is even** is the statement: **For all integers, x, x^2 is even but x is odd**.

T F [(25 DIV 6) - (133 MOD 10)] = 2.

T F If d|(x + y), then d|x and d|y.

T F If A = {*x*,*y*,*z*}, then A × A = {(*x*,*x*),(*x*,*y*),(*x*,*z*),(*y*,*x*),(*y*,*y*),(*y*,*z*),(*z*,*x*),(*z*,*y*),(*z*,*z*)}.

T F If Σ = {0,1}, then $|\Sigma \times \Sigma \times \Sigma| = |\Sigma^3|$.

T F If A ∪ B = C and A and B partition C, then A ∩ B = ∅.

T F For all integers x, if x is odd, then x^2 MOD 8 = 1.

2. (6 pts.) Use the Euclidian Algorithm to find gcd(8398,390)

3. (10 pts.) Show, without using truth tables, that $(\sim p \rightarrow r) \wedge (\sim q \rightarrow r) \equiv \sim (p \wedge q) \rightarrow r$.

4. (4 pts.) Give the converse, inverse, contrapositive, and negation of the universal statement:

For all $x \in N$, if $x^2 = 4$, then x = 2.

5. (10 pts.) Find the Boolean polynomial representing a circuit of four inputs in such a way that if the first three inputs are each the opposite of the last, then current flows out of the circuit.

6. (10 pts.) Use Venn Diagrams to show that: $(A \cap B)^c = A^c \cup B^c$.

7. (40 pts.) Prove 2 of the 4 theorems:

Theorem 1: The product of a rational number and an irrational number is irrational.

Theorem 2: If a and b are *distinct* integers then there is a rational number between them.

Theorem 3: If every integer has a unique representation as the product of prime numbers, then the set of prime numbers is infinite.

Theorem 4: If n,d,q,r are integers with n = dq + r and $0 \le r < d$, then gcd(n,d) = gcd(d,r).