Discrete Structures - Fall 1994 Exam 2

1. Let f:R $\rightarrow$ R be the absolute value function f(x) = |x|, and E be the relation on R: E = {(x,y) | x,y $\in$ R and |x| = |y|}. Show that E is an Equivalence Relation.

2. Use Mathematical Induction to verify: $\sum_{i=0}^{n} 10^i = \frac{10^{n+1}-1}{9}$

3. Rewrite 5 + 5(3) + 5(3^2) + 5(3^3) +...+ 5(3^m) in summation form.

4. Calculate $\sum_{k=1}^{12} \left(\frac{1}{k+1} - \frac{1}{k}\right)$.

5. Let Σ = {0,1} be an alphabet and consider the functions H:$\Sigma^n \times \Sigma^n \rightarrow Z_n$, d:$\Sigma^n \rightarrow Z_n$ and L:$\Sigma^n \rightarrow Z_n$ defined, for any strings s,t in Σ^n as:

H(s,t) = the number of positions where s and t disagree (Hamming Distance Function),
d(s) = the number of 1's in string s (Density Function),
L(s) = the number of characters in string s (Length Function).

Calculate: **(a)** H(10110111,11001101) **(b)** d(10100011) **(c)** L(θ)

6. Rewrite $\sum_{k=1}^{225} (133 - k)$ using the change of variables j = k + 1

7. Consider the sequence: $s_i = s_{i-1} + s_{i-2} + s_{i-3}$ with $s_0 = 1$, $s_1 = 2$, and $s_2 = 4$. Use the *Strong Form of Mathematical Induction* to prove that $s_i \leq 3^i$, $i \geq 3$.

8. For any set A, let n(A) denote the cardinality of A. Fill in the blanks below with one of <, $\leq$, =, $\geq$, or >, to make the statement correct.

(a) If f:A $\rightarrow$ B is a *one-to-one* function then n(A) ___ n(B).

(b) If f:A $\rightarrow$ B is an *onto* function then n(A) ___ n(B).

(c) If f:A $\rightarrow$ B is a *bijective* function then n(A) ___ n(B).

(d) If f:A $\rightarrow$ B is a *onto* function then f(A) ___ B.

9. Let f = {(1,4),(2,3),(3,1),(4,5),(6,7)} and g = {(1,5),(3,7),(4,9),(5,11),(7,13)} be functions.

(a) The domain of f = ___ **(b)** The image of g = ____ **(c)** g°f = ____

10. Let A = {1,2,3,4,5,6,7,8,9,10} and R be the relation of congruence modulo 3 on A; that is, R = {(x,y) | x,y $\in$ A and 3|(x - y)}.

(a) Draw the directed graph of R.

(b) Describe the partition induced on A by the equivalence relation R.