Discrete Structures - Fall 1994 - Examination 1

1. (20 pts.) Circle T if the statement is true or F if the statement is false.

T F $(\mathbb{R} \cap \mathbb{Q}) \cup \mathbb{Z} = \mathbb{Z}$.

T F If $\mathbb{P}(A)$ is the Power Set of A and $\varnothing$ is the Empty Set, then $\varnothing \subset \mathbb{P}(A)$ and $\varnothing \in \mathbb{P}(A)$.

T F The negation of the statement All students who take CMSC 203 and pass will get a Computer Science degree is the statement All students who take CMSC 203 and pass will not get a Computer Science Degree.

T F The statements If I pass CMSC 203, then I will get a good job and If I do not get a good job, then I did not pass CMSC 203 are logically equivalent.

T F If A = {x,y,z}, then (z,z,y) $\in$ A $\times$ A.

T F If Σ = {x,y,z}, then (zz,y) $\in \Sigma^2 \times \Sigma$.

T F The set of the integers is partitioned by the set of positive integers and the set of negative integers .

T F (Show work!) $\left[\left[(33 \bmod 6) - \left\lceil \frac{126}{102} \right\rceil\right] \lfloor (-13.2) + 6 \rfloor\right] \bmod 5 = 2$.

T F If n and d are positive integers, then n = d(n div d) + (n mod d).

T F If A is a set with 8 elements, and a $\in$ A, then A has 128 subsets containing a.

2. (6 pts.) Given the statement All kids who run fast win races,
its converse is _____; its inverse is_____; its contrapositive is_____.

3. (10 pts.) Show, without using truth tables, that $p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$.

4. (4 pts.) Write the negation of the universal statement: For all $x \in \mathbb{Z}$, if $x^2 = 9$, then x = 3 or x = –3.

5. (10 pts.) Find the <u>Truth Table</u> and <u>Boolean Polynomial</u> representing a circuit of three switches controlling a light bulb in such a way that if the first and third switches are both ON or both OFF, the bulb is ON.

6. (10 pts.) Using the Properties of Sets, show that: (A – B) – C = (A – C) – B.

7. (40 pts.) Prove 2 of the 4 theorems:
Theorem 1: If every integer has a unique representation as the product of primes, then there are an infinite number of primes.

Theorem 2: If any integer n can be expressed as 3k, 3k + 1, or 3k + 2, then the product of three consecutive integers is divisible by 3..

Theorem 3: If $q \in \mathbb{Q}$ and $r \in \mathbb{Q}$, then $qr \in \mathbb{Q}$.

Theorem 4: For all integers a,b,c, and d, if a | (b + c) and a | d, then a | (b + c + d).