**Exam 2 Discrete Structures CMSC 203 Fall 2008**

**1.** Circle **T** if the corresponding statement is True or **F** if it is False.

**T** **F** GCD(1,0) = 0.

**T** **F** For every recursive algorithm, there is an equivalent sequential algorithm.

**T** **F** In general, sequential algorithms use memory less efficiently than their equivalent recursive representations.

**T** **F** If $a$ and $b$ are positive integers, then $a = b(a \text{ div } b) + (a \bmod b)$.

**T** **F** $1 + 2 + 4 + 8 + 16 + \ldots + 2^{24} = 2^{25} + 1$.

**T** **F** Algorithms whose order is $O(n\log n)$ are less efficient than those of order $O(n^2)$.

**T** **F** Given two algorithms, $f(n)$ and $g(n)$, $O(f + g) = \max(O(f), O(g))$.

**T** **F** GCD( $p,q$) = GCD($q$, $p \bmod q$).

**2.** Let $\{a_n\}$ and $\{b_n\}$ be the sequences defined, for $n \geq 0$, by:

$a_n = (n + 1)$, and $b_n = (n + 2)$. Find $c_1$, $c_2$, $c_3$, and $c_4$ when $c_n = (a_n)(b_n) + n$.

**3.** Write out the Division Algorithm and trace its steps to calculate (54 MOD 12).

**4. (a)** Give a Recursive Definition for the even Natural numbers.

**(b)** Find the Big-Oh of the algorithm with complexity: $(n^3 + 3)(n^6 + 6) + [3n^3 + 2n^2(\log n)]$.

**5. (a)** Given $a = 2^3 3^0 5^2 7^4 11^0 13^1 17^3 19^0 23^0 29^9 31^2$.

and $b = 2^1 3^2 5^2 7^0 11^1 13^3 17^0 19^1 23^2 29^1 31^4$, then

GCD($a$,$b$) = _____ and LCM($a$,$b$) =_____.

**(b)** List out the search intervals in applying the Binary Search algorithm to find 14 in the list:

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Pass 1:

Pass 2:

Pass 3:

Pass 4:

**6.** Prove ONE of the TWO Theorems below using Mathematical Induction.

<u>Theorem 1</u>: For all integers $n \geq 1$, $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

<u>Theorem 2</u>: If $a_0 = 5$, $a_1 = 10$, and $a_2 = 15$, then $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ is a multiple of 5, for all $n \geq 3$.

**7.** Prove ONE of the TWO Theorems below:

<u>Theorem 1:</u> If $a$, $b$, and $m$ are positive integers, with ($a \bmod m$) = ($b \bmod m$), then $a \equiv b \bmod m$.

<u>Theorem 2:</u> The difference of the squares of successive positive integers is odd.

**8.** Prove ONE of the TWO Theorems below by Contradiction or Contraposition.

<u>Theorem 1:</u> There is no largest Natural number.

<u>Theorem 2:</u> If $n$ is an integer and $n^2$ is odd, then $n$ is odd.