SAMPLE Exam 1 - Fall 2006 - Discrete Structures

1. Circle **T** for True or **F** for False as they apply to the following statements:

T F All compound statements are either a tautology or contradiction.

T F The argument "*All dogs bark* and *Ralph barks*, therefore *Ralph is a dog*" is an example of Converse Error.

T F Every set is a subset of itself.

T F If a is an element of the set A, then half the subsets of A contain a.

T F For any sets A and B, $(A \cup B) \subseteq (A \cap B)$.

T F The converse and inverse forms of a conditional statement are logically equivalent.

T F If the set $|A| = 4$ then $|A \times A \times A \times A| = 256$.

T F The Domain and Range of bijective functions are the same set.

T F The Cardinality of the Rationals and the Reals is the same.

T F If s is a binary string of length 8 and $t = (s \oplus 11111111)$, then $H(s,t) = 8$.

2. Find the truth table for the compound statement: $[(p \leftrightarrow q) \oplus r] \vee [p \wedge (\neg q \rightarrow r)]$

3. Find the related forms for the statement: **For all Integers, x, if x is odd, then x^2 is odd.**

CONVERSE: _____ NEGATION: _____

4. Draw a graph for a function, f:{1, 2, 3, 4}$\rightarrow$ {w, x, y, z}, that is: **(a)** onto; **(b)** one-to-one.

5. Show that the function $f : \mathbf{R} \rightarrow \mathbf{R}$ defined as $f(x) = 7x - 3$ is a bijection.

6. Calculate the following (assuming all strings are from the alphabet {0, 1}):

(a) $\ell(1001001001)$

(b) d(010011000111)

(c) H(11011101 , 00110011)

(d) $(\lfloor 8.1 \rfloor - 5)(\lceil -5.1 \rceil + 8)$

7. (a) Let $f = \{(0,35), (1,55), (2,15), (3,25), (4,45)\}$

and $g = \{(15,400), (25,100), (35,200), (45,300), (55,500)\}$.

Show that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

(b) Find the Inverse of the function of $h = \{(1,2), (2,3), (3,4), (4,5), (5,6)\}$.

8. Use the logic of valid arguments to determine whether or not we can deduce t:

$$s \rightarrow \sim q$$
$$\neg p \rightarrow (r \vee s)$$
$$\neg p \wedge q$$
$$r \rightarrow t$$

9. Use the Properties of Sets to verify for any sets A, B, C, and D

$$(A^c \cup B \cup C \cup D)^c = [(A - B) - C] - D.$$