CMSC 203 - Discrete Structures - FINAL EXAMIMATION - Part Two - Fall 2002

1. Construct the truth table for the compound proposition: $[q \vee (\neg q \rightarrow \neg p)] \leftrightarrow (p \rightarrow \neg q)$

2. Use the Laws of Logic to verify: $p \wedge (p \vee q) \equiv p$.

3. What is the negation of the quantified statement: Every dog that chases parked cars has a flat nose.

4. Find A × B for the sets A = { ∅, {1} } and B = { {0}, ∅ }

5. Show that the function f: **R** → **R** given by $f(x) = 9x + 7$ is One-To-One and Onto.

6. Let the function f: **R** → **R** be $f(x) = x + 3$ and the function g: **R** → **R** be $g(x) = x^2 + 2x + 1$. Find:
(a) $f(g(x))$ **(b)** $g(f(x))$

7. (a) Find the big-O estimate for the function $F(n) = (n^4 \log n + n^6 + 6n)(n^2 + 1)$.
(b) Find the Hamming Distance between 100110011001 and 101100111000.
(c) Using the Euclidean Algorithm, find gcd(1024,120).

8. If a, b, and c are integers with $a = b + c$, prove that $\gcd(a,b) = \gcd(b,c)$.
(Hint: if $\gcd(a,b) \leq \gcd(b,c)$ and $\gcd(a,b) \geq \gcd(b,c)$, then $\gcd(a,b) = \gcd(b,c)$).

9. Use the following Theorem:
<u>THEOREM:</u> For all integers, *n*, and prime numbers, *p*, if *p* divides n^2, then *p* divides *n*.
to prove by the Method of Contradiction that $\sqrt{3}$ is irrational.

10. Let a be a Real Number not equal to 0 or 1. Use Math Induction to prove for all integers $n \geq 0$,

$$\sum_{i=0}^{n} a^i = \frac{a^{n+1} - 1}{a - 1}.$$

11. (a) How many ways can Art, Betty, Chad, Deb, Ed, Fran, Glen, Helen, Ivan, and Jane form a line if Ed cannot be immediately between Betty and Ivan?

(b) The Mars Candy Company sells bags of M&M candies with 60 pieces candy colored from 8 different colors in them. How many different bags can they produce if they want at least 1 of the first color, 2 of the second, color, 3 of the third color, 4 of the fourth color, ..., and 8 of the eighth color?