Discrete Structures - Fall 2001- Examination 3

1. How many distinct words can I create from the letters of the words:
a. COLUMBIA **b.** PENNSYLVANIA

2. How many distinct ways can I arrange 15 people in a circle if a certain pair MUST sit next to one another?

3. How many ways can a state issue 8-character license plates if the characters are one of 52 letters (upper and lower case) or 10 digits, and the state wants:
a. the characters to alternate letter, digit, letter, digit, etc., with the first character a letter?
b. the first character to be a digit and the last two to be letters?

4. Given the alphabet $\{w, x, y, z\}$, how many 20-long strings have 3 w' s, 4 x's, and 5 y's?

5. How many ways can I arrange 12 Math, 15 Computer, and 10 Chemistry books on a shelf...
a. ...if all the books of the same type must be grouped together?
b. ...if I all the books of the same type must be grouped together and the Math books must be in the middle?

6. The ACME Candy company makes 47 different varieties of candy.
a. How many ways can they create gift boxes containing 30 pieces of candy?
b. How many ways can they create gift boxes containing 75 pieces of candy, if at least one piece of each type must be in the box?

7. Use the iterative method to find the particular solution of the recurrence relation:

$$s_n = 3s_{n-1} + 7 \text{ with } s_0 = 1$$

8. Find the characteristic polynomial to the recurrence relation:
a. $s_n = 9s_{n-1} - 9s_{n-2}$ **b.** $s_n = 9s_{n-3} - 9s_{n-5}$

9. Find the general solution to the recurrence relation
a. $s_n = -2s_{n-1} + 99s_{n-2}$ **b.** with characteristic poly. equation: $(x - 7)^5 = 0$.

10. Find the particular solution to the recurrence relation whose general solution is:

$$s_n = A(2^n) + B(3^n), \quad \text{subject to } s_0 = 3 \text{ and } s_1 = 1.$$