CMSC 203 Section 0201: Homework1 Solution

1. **Exercise Set 1.1**

    Problem 15: Write truth table for the statement forms: *(5 points)*

    ~(p ^ q) V (p V q)

    Truth Table:

p	q	p ^ q	p V q	~(p ^ q)	~(p ^ q) V (p V q)
T	T	T	T	F	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	F	T	T

    Problem 28: *(10 points)*

    Determine which of the following pair of statement are logically equivalent

    (r V p) ^ ((~r V (p^q)) ^ (r V q)) and p ^ q

    Truth Table:

p	q	r	~r	p ^ q	~r V (p^q)	r V q	r V p	(~r V (p^q)) ^ (r V q)	9 ^ 8
1	2	3	4	5	6	7	8	9	10
T	T	T	F	T	T	T	T	T	T
T	T	F	T	T	T	T	T	T	T
T	F	T	F	F	F	T	T	F	F
T	F	F	T	F	T	F	T	F	F
F	T	T	F	F	F	T	T	F	F
F	T	F	T	F	T	T	F	T	F
F	F	T	F	F	F	T	T	F	F
F	F	F	T	F	T	F	F	F	F

    Since values in the column 5 and column 10 are same the give two statements are logically equivalent.

    Problem 49: *(10 points)*

    Use theorem 1.1.1 to prove the logical equivalence of the following statements,

    ~(p V ~q) V (~p ^ ~q) ≡ ~p

    Sol:

    ~(p V ~q) V (~p ^ ~q) ≡ ~p

    (~p ^ ~(~q)) V (~p ^ ~q)   (De Morgan's law)

(~p ^ q) V (~p ^ ~q)	(Double negative law)
~p ^ (q V ~q)	(Distributive law)
~p ^ t	(Negation law)
~p	(Identity law)

Hence proved

2. Exercise Set 1.2

Problem 6:   *(10 points)*

Construct the truth table for the statements

(pVq) V (~p^q) → q

p	q	~p	p V q	~p ^ q	(p V q) V (~p ^ q)	(p V q) V (~p ^ q) → q
T	T	F	T	F	T	T
T	F	F	T	F	T	F
F	T	T	T	T	T	T
F	F	T	F	F	F	T

Problem 18:   *(15 points)*

Write each of the following three statements in the symbolic form and determine which pairs are logically equivalent

a.   If it walks like a duck and it talks like a duck, then it is a duck
b.   Either it does not walk like a duck or it does not talk like a duck, or it is a duck
c.   If it does not walk like a duck and it does not talk like a duck, then it is not a duck

Sol:

Let:

p = it walks like a duck
q = it talks like a duck
r = it is a duck

Then given statements can be represented as:

a.   (p ^ q) → r
b.   ~p V ~q V r
c.   (~p ^ ~q) → ~r

The truth tables for each of the above statements are as below:

a. (p ^ q) → r

p	q	r	p ^ q	(p ^ q) → r
T	T	T	T	T
T	T	F	T	F

T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

b. ~p V ~q V r

p	q	r	~p	~q	~p V ~q	~p V ~q V r
T	T	T	F	F	F	T
T	T	F	F	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

c. (~p ^ ~q) → ~r

p	q	r	~p	~q	~r	~p ^ ~q	(~p ^ ~q) → ~r
T	T	T	F	F	F	F	T
T	T	F	F	F	T	F	T
T	F	T	F	T	F	F	T
T	F	F	F	T	T	F	T
F	T	T	T	F	F	F	T
F	T	F	T	F	T	F	T
F	F	T	T	T	F	T	F
F	F	F	T	T	T	T	T

Since the truth tables for statements a and b are same the two statements are equivalent.

Problem 48:   *(5 points)*

Rewrite the statement in the if-then form

A necessary condition for this computer program to be correct is that it does not produce error messages during translation:

Sol:

If the computer program produces error messages during translation then it is not correct.

3. **Exercise Set 1.3**

   Problem 10:   *(10 points)*

   Use truth tables to determine if the argument forms are valid

   p → r
   q → r
   hence,
   p V q → r

| p | q | r | p V q | PREMISES | | CONCLUSION |
				p → r	q → r	(p V q) → r
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	F	T	T	T
F	F	F	F	T	T	T

   Since for all the critical rows when the premises are true the corresponding conclusion is also true. Hence the argument form is valid.

   Problem 13 b: *(10 points)*

   Use truth tables to show that the following forms of argument are invalid:

   p → q
   ~p
   hence,
   ~q
   (inverse error)

| p | q | PREMISES | | CONCLUSION |
		~p	p → q	~q
T	T	F	T	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

   Since for the third row when the premises are true and the conclusion is false, hence the given argument is invalid.

   Problem 28:   *(5 points)*

   Use symbols to write the logical form of each argument. If the rule is valid then identify the rule of inference that guarantees its validity, otherwise state whether converse or inverse error is made.

Sol:

Let,

p = If there are as many rational numbers as irrational numbers
q = The set of all irrational numbers is infinite

Then given statements can be written as,

p → q
q
hence,
p

The above set of arguments is not valid since it exhibits the converse error.

Problem 42:   *(20 points)*

Use the valid argument forms listed in Table 1.3.1 to deduce the conclusion from the premises, giving a reason for each step.

a.   p V q
b.   q → r
c.   p ^ s → t
d.   ~r
e.   ~q → u ^ s
f.   Hence,
     t

Sol:

We have,

q → r &
~r
Therefore,
~q   **(Modus Tollens)**

Now,
p V q &
~q
Therefore,
p   **(Elimination)**

Now,
~q → u ^ s &
~q
Therefore,
u ^ s   **(Modes Ponens)**

Now,
u ^ s
Therefore,
s   **(Specialization)**

Now,
p &
s
Therefore,
p ^ s   **(Conjunction)**

Now,
p ^ s → t &
p ^ s
Therefore,
t   **(Modes Ponens)**

Hence proved,