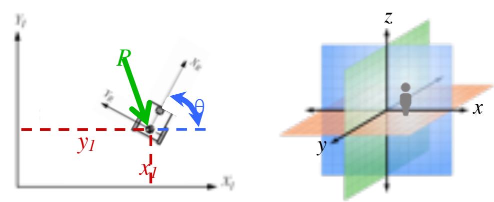
Kinematics: overview, transforms, and wheels



Kinematics

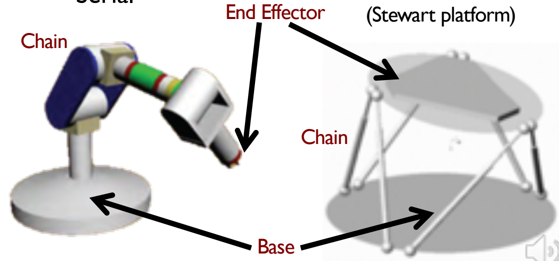
- What is kinematics?
  - The study of the motion of objects.
  - The study of the **geometrically possible motion** of a body or system of bodies
    - Regardless of causes and effects of motion
- Movement determines the (eventual) position and orientation of the robot
  - Mobile: position and orientation wrt. some initial frame
  - Manipulator: position and orientation of end effector
- But… what are we trying to do??
  - **Get a robot where we want it to be!**

Review: Kinematic Models

- Models **how a system can move in the world**
  1. With respect to the world and its own parts
  2. Configuration: where are all the points on the system?
- Manipulators: links, joints, base
  - Manipulator links from a chain
  - Serial or parallel (mostly)
- Mobile robots: possible x/y/z movement
  - Omni wheels ≠ wheels ≠ flying

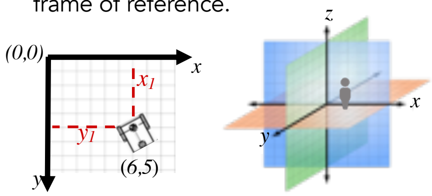
Review: Kinematic Chains

Multiple links and joints are called a *kinematic chain.*



Review: Frames of Reference

- What's a frame of reference?
  - A coordinate system + point(s) to locate and orient it
  - Usually x, y, and sometimes z coordinates, plus origin
- Things move with respect to some frame of reference.



en.wikipedia.org/wiki/Frame_of_reference, study.com/academy/lesson/coordinate-system-in-geometry-definition-types.html

Kinematics and IK

- Need **position** and **orientation** of the robot
  - Mobile: with respect to an arbitrary initial frame
  - Manipulator: position and orientation of **end effector**
- Forward kinematics: from parameters to configuration
  - A configuration is ________________
- Inverse Kinematics (IK): from a desired configuration to parameters that make it so

Forward Kinematics

- **Find** position and orientation **from** parameters
  - Mobile: robot center
  - Manipulator: end effector
- Manip. Forward Kinematics (angles to position)
  - **Given:**
    - Kinematic model *plus*
    - Angle/displacement of each joint
  - **Find:**
    - The position of any point
      - E.g.: Paintbrush is at *these* coordinates, pointed *this* way

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Inverse Kinematics (IK)

- Find parameters from position a
  - Mobile: robot center
  - Manipulator: end effector
- Manip. **Inverse** Kinematics (posi
  - **Given:**
    - Parameters and kinematics mod
    - Desired position/orientation of robot
  - **Find:**
    - Parameters: angle/displacemen obtain that position

Why?
We have *direct* control over joints.
We have *indirect* control over robot's position in the world.
If we want the paintbrush here…

we need to know what to tell *joints* to do.

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Position and Orientation

What do these mean for…	Position: Where is it?	What's its orientation?
Mobile Robot	On an $\{x,y\}$ plane 	Heading θ 
Manipulator	In some $\{x,y,z\}$ space 	$\{r/p/y\}$ of end effector 

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Mobile Robot Kinematics

- Description of mechanical behavior of the robot for design and control
  - Similar to manipulator kinematics
- However, mobile robots can move unbounded with respect to the environment
  - No direct way to measure robot's position
  - Position must be integrated over time
  - Leads to inaccuracies of the position (motion) estimate

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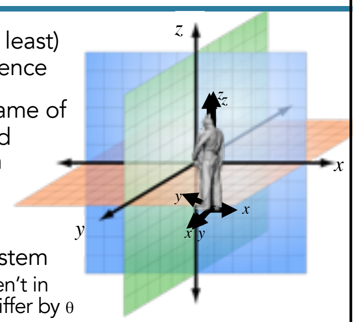
Global + Local FoR

- We always have (at least) two frames of reference
- **Global** (or initial) frame of reference: the world the system exists in
- **Local** (or robot) frame of reference: grounded in the system
  - If she turns, they aren't in agreement – they differ by θ



- **Transform** coordinates from one to the other

en.wikipedia.org/wiki/Frame_of_reference, study.com/academy/lesson/coordinate-system-in-geometry-definition-types 11

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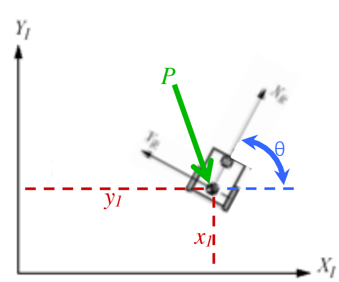
Kinematics

Transforms & Wheels



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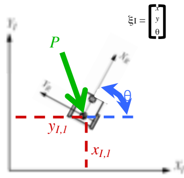
Mobile Kinematics: Concepts

- Forward Kinematics:
  - **Parameters → Configuration**
- Inverse Kinematics (IK):
  - **Configuration → Parameters**
  - I want to be in this configuration. What motions should I make?
- Mobile **configuration** = position and orientation with respect to an arbitrary initial frame I
- Understanding mobile robot motion starts with **understanding constraints on the robot's mobility**



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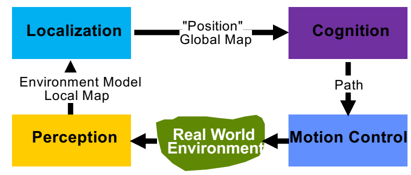
Wheeled Motion Control

- Requirements for controlling motion:
  - Kinematic / dynamic models of the robot
  - Model of interaction between the wheel and the ground
  - Definition of required motion
    - What speed and position controls are there? Are possible?
  - A control policy that satisfies the requirements



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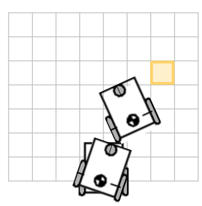
What We're Trying to Do

- Sequence of events:
  - Power on: position = (0,0), orientation = due north
  - Rotate 15° right
  - Move forward 2 meters
  - Observe obstacle
  - Rotate 30° left
  - Move forward 1 meter
- Position – (?,?), orientation = ?°
- Where's the obstacle?



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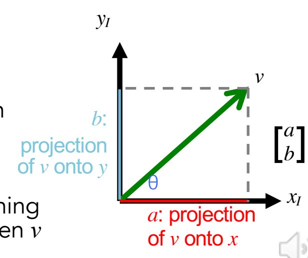
Projections

- A projection of a vector v onto an axis is the amount of change along that axis along v
- Here:
  - $a = \Delta x_I$
  - $b = \Delta y_I$
- This is the change in position in that axis
- So now we can talk about where something is on x_I, y_I* axes, given v



*The initial = global = map axes and coordinates

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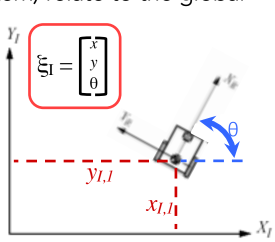
Specifying Transforms

- How does a robot (or system) relate to the global frame of reference?
  - Configuration = position and orientation
- Position: x,y coordinates $x_{I,t}$ and $y_{I,t}$
  - I = initial (global)
  - t = timestep
- Orientation: θ
  - Angle between robot's coordinate system and initial coordinate system



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Mobile Position & Orientation

Frames of reference:

$\{X_I, Y_I\}$: Global

$\{X_R, Y_R\}$: Robot

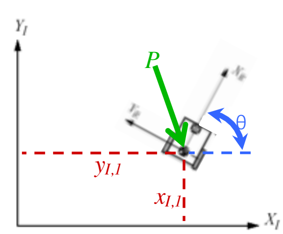
Robot: point P

Position (of P):

$\{x_{I,I}, y_{I,I}\}$

Heading:

$\{\theta\}$: I ∠ R



$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

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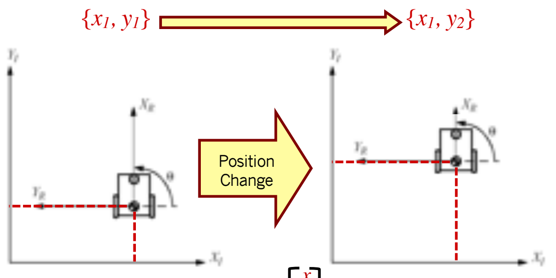
Mobile Position Change



$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

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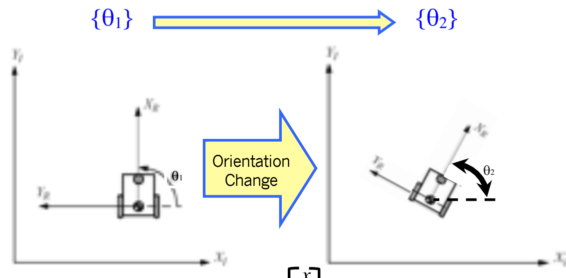
Mobile Orientation Change



$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

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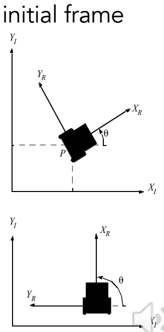
Mapping Between Frames

- Representing robot in an arbitrary initial frame
  - Initial frame: $\{X_I, Y_I\}$
  - Robot frame: $\{X_R, Y_R\}$
  - Robot: $\xi_I = [x \;\; y \;\; \theta]^T$
    - Just the transpose notation
- Goal
  - Map motions from global reference frame to local reference frame (and vice versa)

$\Delta\{X_R, Y_R\} \rightarrow \Delta\{X_I, Y_I\}$



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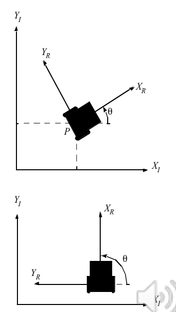
Mapping Between Frames

- Global reference frame ⬌ local reference frame

  $\{X_I, Y_I\} \Leftrightarrow \{X_R, Y_R\}$

  - Map motion from **axes** of one to **axes** of the other
    - This mapping depends on current pose
- How do you do this mapping?
  - How do you perform a rotation in Euclidean spaces?



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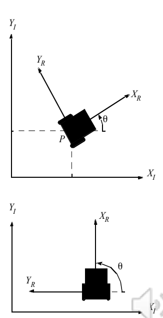
Mapping Between Frames

- With a **rotation matrix**
- Any point $<x,y>$ in space (aka $\begin{bmatrix} x \\ y \end{bmatrix}$) can be multiplied by some matrix…
  - (spoiler: it's usually) $R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
- The result is the coordinates in the **other** frame, rotated by θ around z.
- This matrix rotates points in the x-y plane counter-clockwise, through θ, around the origin.



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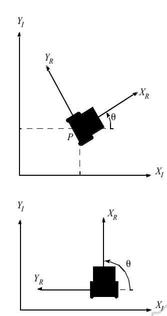
Mapping Between Frames

- Global reference frame ⬌ local reference frame

  $\{X_I, Y_I\} \Leftrightarrow \{X_R, Y_R\}$

  - Map motion from **axes** of one to **axes** of the other
    - Mapping depends on current pose
- Use the *orthogonal reference frame**:

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



**this one rotates around z* 25

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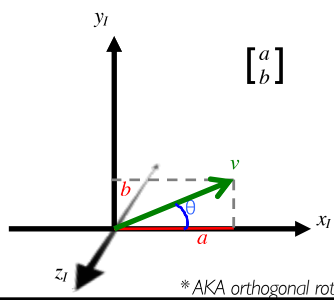
The Z Rotation Matrix



*AKA orthogonal rotation matrix 26

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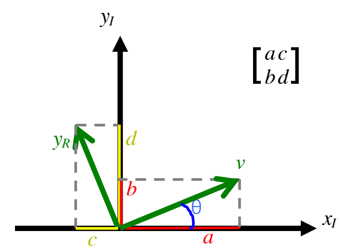
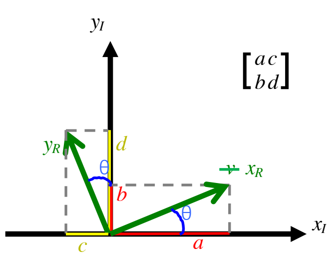
The Z Rotation Matrix*



*AKA orthogonal rotation matrix 27

27

The Z Rotation Matrix*



*AKA orthogonal rotation matrix 28

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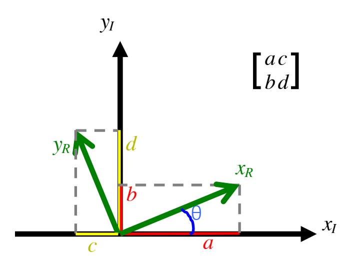
The Z Rotation Matrix

- If we assume frame axes are of length 1…



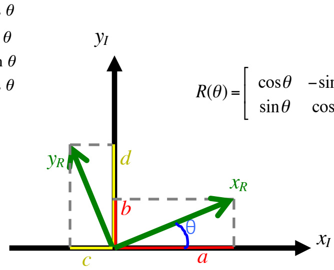
*AKA orthogonal rotation matrix 29

29

The Z Rotation Matrix

- If we assume frame axes are of length 1
  - $a = \cos\theta$
  - $b = \sin\theta$
  - $c = -\sin\theta$
  - $d = \cos\theta$

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$



*AKA orthogonal rotation matrix 30

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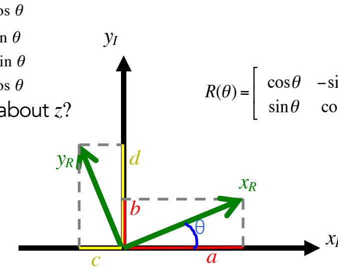
The Z Rotation Matrix

- If we assume frame axes are of length 1
  - $a = \cos\theta$
  - $b = \sin\theta$
  - $c = -\sin\theta$
  - $d = \cos\theta$
- What about z?

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$



*AKA orthogonal rotation matrix 31

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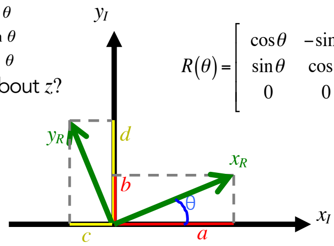
The Z Rotation Matrix

- If we assume frame axes are of length 1
  - $a = \cos\theta$
  - $b = \sin\theta$
  - $c = -\sin\theta$
  - $d = \cos\theta$
- What about z?

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



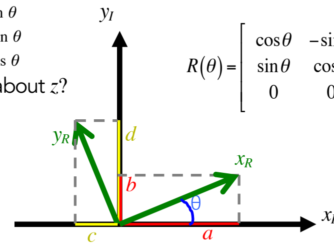
*AKA orthogonal rotation matrix 32

32

The Z Rotation Matrix

- If we assume frame axes are of length 1
  - $a = \cos\theta$
  - $b = \sin\theta$
  - $c = -\sin\theta$
  - $d = \cos\theta$
- What about z?

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



*AKA orthogonal rotation matrix 33

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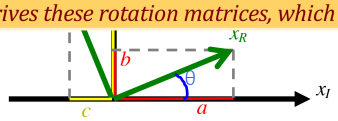
The Z Rotation Matrix

- If we assume frame axes are of length 1
  - $a = \cos\theta$
  - $b = \sin\theta$

Now Watch
https://youtu.be/lVjFhNv2N8o
She goes through the same material, but a bit slower; she also derives these rotation matrices, which can help.



*AKA orthogonal rotation matrix 34

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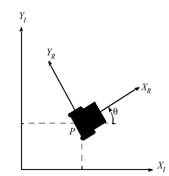
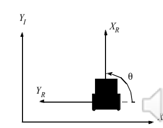
Mapping Between Frames

- Rotation matrices (in Euclidean space)

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_R \\ y_R \end{bmatrix} = \begin{bmatrix} x_I \\ y_I \end{bmatrix}$$

- Rotates points in the x-y plane counter-clockwise, through θ, around the origin.
- To use R, the position of each point is represented by a **vector**.
- A rotated vector is then obtained with matrix multiplication.

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Orthogonal Rotation Matrix

- This mapping function is called

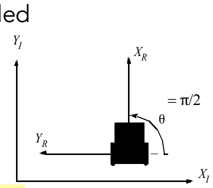
$R(\theta)\,\dot{\xi}_I$

because it depends on θ.

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Z rotation matrix

$\dot{\xi}_R = R(\pi/2)\,\dot{\xi}_I$



cos(pi/2)=0
sin(pi/2) =1
tan(pi/2) =infty
cot(pi/2) =0
csc(pi/2)=1
sec(pi/2)=infty

- **Example:**

$$R(\pi/2) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Why?

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Orthogonal Rotation Matrix

- This mapping function is called

$R(\theta)\,\dot{\xi}_I$

Now Watch
https://youtu.be/0YuoPTRVzxY
For another (brief) explanation and an example.

Z rotation matrix

$\dot{\xi}_R = R(\pi/2)\,\dot{\xi}_I$

cos(pi/2)=0
sin(pi/2) =1
tan(pi/2) =infty
cot(pi/2) =0
csc(pi/2)=1
sec(pi/2)=infty

- **Example:**

$$R(\pi/2) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Why?

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Next time…

- Whoosh!
- Class: let's actually do some of this!
- Next lecture: calculating movement using displacement vectors
- Some of the kinematics of 3D movement

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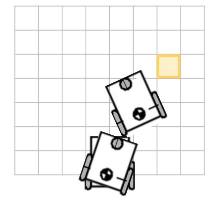
Remember this?

- Power on: position = (0,0), orientation = due north
- Rotate 15° right
- Move forward 2 meters
- Observe obstacle
- Rotate 30° left
- Move forward 1 meter
- Position – (?,?), orientation = ?°



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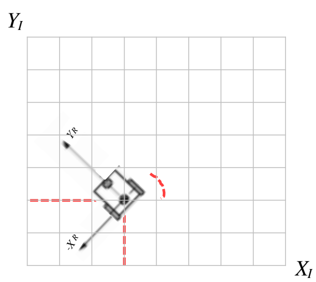
Rotation matrices, ex 1

- What is the current configuration of the robot, by eye?



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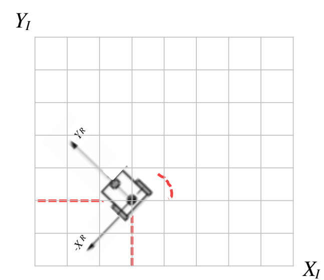
Rotation matrices, ex 1

- What is the current configuration of the robot, by eye?

$\xi_I =$

- *Configuration in the initial frame is denoted by* ξ_I



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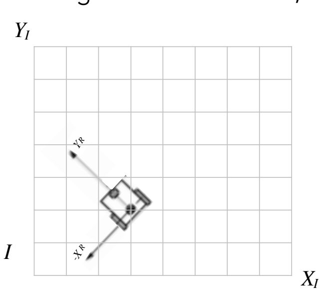
Rotation matrices, ex 1

- What is the current configuration of the robot, by eye?

$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

- *Configuration in the initial frame is denoted by* ξ_I
- x, y, *and* θ *denote the coordinates and rotation in* I



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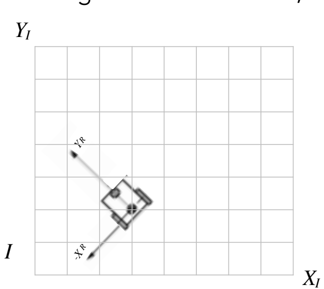
Rotation matrices, ex 1

- What is the current configuration of the robot, by eye?

$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 45^\circ \end{bmatrix}$$

- *Configuration in the initial frame is denoted by* ξ_I
- x, y, *and* θ *denote the coordinates and rotation in* I
- *This looks right*



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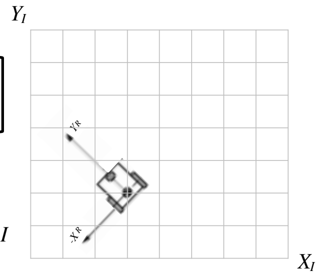
Rotation matrices, ex 1

- What is the current configuration of the robot, by eye?

$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 45^\circ \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ \pi/4 \end{bmatrix}$$



- *Configuration in the initial frame is denoted by* ξ_I
- x, y, *and* θ *denote the coordinates and rotation in* I
- *This looks right*
- *It's always easier to use radians*

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Discussion

- Why is it 45° and not 315°?
  - Because this is the *counterclockwise* rotation matrix
- Why does the robot show axis $-X_R$ instead of X_R?
  - To make it easier to line up with X_I
  - Because I don't have space to draw the arrow pointing the other way

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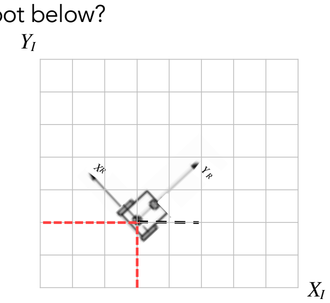
Rotation matrices, ex 2

- What is the (solved) instantaneous rotation matrix of the robot below?



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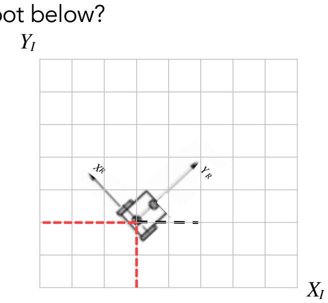
Rotation matrices, ex 2

- What is the (solved) instantaneous rotation matrix of the robot below?

$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} =$$

- *First, the configuration*



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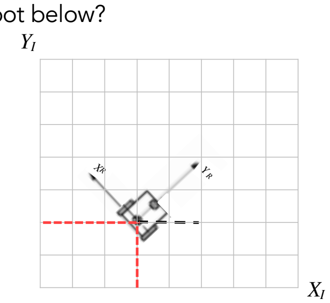
Rotation matrices, ex 2

- What is the (solved) instantaneous rotation matrix of the robot below?

$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 315^\circ \end{bmatrix}$$

- *First, the configuration*
- *This looks right*
  - *Why is it 315°?*



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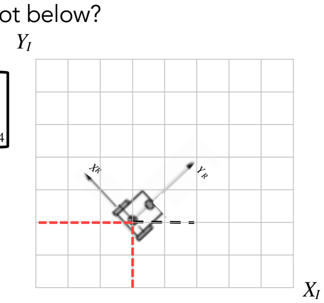
Rotation matrices, ex 2

- What is the (solved) instantaneous rotation matrix of the robot below?

$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 315^\circ \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 7\pi/4 \end{bmatrix}$$

- *First, the configuration*
- *This looks right*
  - *Why is it 315°?*
- *Always easier in radians*



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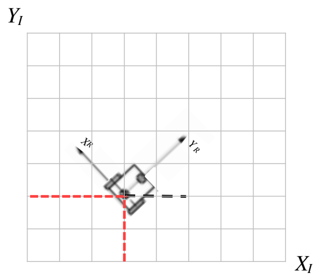
Rotation matrices, ex 2

- What is the (solved) instantaneous rotation matrix of the robot below?

$$\xi_I = \begin{bmatrix} 3 \\ 2 \\ 7\pi/4 \end{bmatrix}$$

- *Okay, how do we get a matrix from this vector?*



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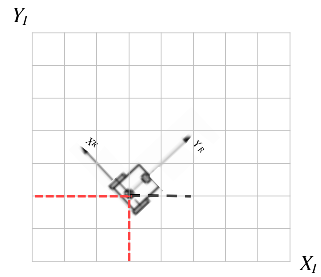
Rotation matrices, ex 2

- What is the (solved) instantaneous rotation matrix of the robot below?

$$\xi_I = \begin{bmatrix} 3 \\ 2 \\ 7\pi/4 \end{bmatrix}$$

- *Okay, how do we get a matrix from this vector?*
- *We rotated counterclockwise around z, so…*



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Rotation matrices, ex 2

- What is the (solved) instantaneous rotation matrix of the robot below?

$$\xi_I = \begin{bmatrix} 3 \\ 2 \\ 7\pi/4 \end{bmatrix}$$

- *Okay, how do we get a matrix from this vector?*
- *We rotated counterclockwise around z, so…*
- *(Let's make some space)*

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Rotation matrices, ex 2

- What is the (solved) instantaneous rotation matrix of the robot below?

$$\xi_I = \begin{bmatrix} 3 \\ 2 \\ 7\pi/4 \end{bmatrix} \qquad R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- *We apply the z rotation matrix*

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53

Rotation matrices, ex 2

- What is the (solved) instantaneous rotation matrix of the robot below?

$$\xi_I = \begin{bmatrix} 3 \\ 2 \\ 7\pi/4 \end{bmatrix}, \quad R(\theta) = \begin{bmatrix} \cos{}^{7\pi}/_4 & -\sin{}^{7\pi}/_4 & 0 \\ \sin{}^{7\pi}/_4 & \cos{}^{7\pi}/_4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- *We apply the z rotation matrix*
- *Plug in theta…*

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Rotation matrices, ex 2

- What is the (solved) instantaneous rotation matrix of the robot below?

$$\xi_I = \begin{bmatrix} 3 \\ 2 \\ 7\pi/4 \end{bmatrix}, \quad R(\theta) = \begin{bmatrix} \cos{}^{7\pi}/_4 & -\sin{}^{7\pi}/_4 & 0 \\ \sin{}^{7\pi}/_4 & \cos{}^{7\pi}/_4 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -0.25 & 0 & 0 \\ 0 & -0.25 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- *We apply the z rotation matrix*
- *Plug in theta…*
- *This is it.*

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55

Rotation matrices, ex 2

- What is the (solved) instantaneous rotation matrix of the robot below?

$$\xi_I = \begin{bmatrix}3\\2\\7\pi/4\end{bmatrix}, \quad R(\theta) = \begin{bmatrix}\cos 7\pi/4 & -\sin 7\pi/4 & 0\\ \sin 7\pi/4 & \cos 7\pi/4 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}-0.25 & 0 & 0\\ 0 & -0.25 & 0\\ 0 & 0 & 1\end{bmatrix}$$

- *We apply the z rotation matrix*
- *Plug in theta…*
- *This is it.*

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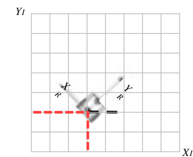
Rotation matrices, ex 2

- What is the (solved) instantaneous rotation matrix of the robot below?

$$\xi_I = \begin{bmatrix}3\\2\\7\pi/4\end{bmatrix}, \quad R(\theta) = \begin{bmatrix}\cos 7\pi/4 & -\sin 7\pi/4 & 0\\ \sin 7\pi/4 & \cos 7\pi/4 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}-0.25 & 0 & 0\\ 0 & -0.25 & 0\\ 0 & 0 & 1\end{bmatrix}$$

- *We apply the z rotation matrix*
- *Plug in theta…*
- *This is it.*
- *The instantaneous rotation matrix of this guy:*



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Rotations matrices, ex 3

- So far we have been working with the z-axis orthogonal rotation matrix, derived:
  - Here: https://youtu.be/lVjFhNv2N8o
  - Here: https://youtu.be/OYuoPTRVzxY
  - Here (new): https://youtu.be/8XRvpDhTJpw
- What if we want to rotate something around another axis, say, x?
  - "Pitch"

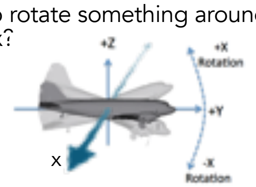

Image: http://www.dirsig.org/docs/new/ppd.html

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Rotations matrices, ex 3

- Consider: what's the instantaneous rotation matrix for this robot?
- How would you **derive** the orthogonal rotation matrix for going counterclockwise around X?
- Or some arbitrary axis?

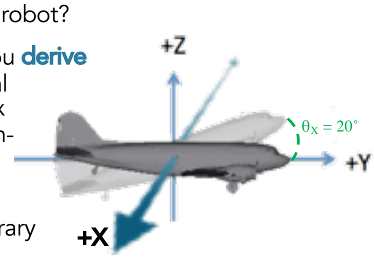

Image: http://www.dirsig.org/docs/new/ppd.html

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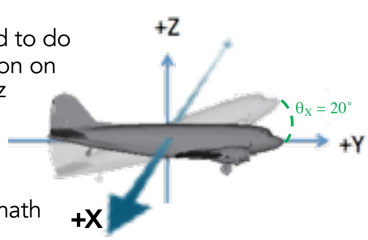
Rotations matrices, ex 3

- What's the instantaneous rotation matrix for this robot?
- You won't need to do such a derivation on this week's quiz
- But it's a good idea to consider the source of the math
  - In general!



- Solution: https://youtu.be.com/wg9bl8-Qx2Q

Image: http://www.dirsig.org/docs/new/ppd.html

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