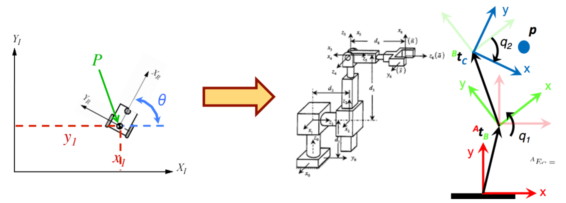
Kinematics and IK

Manipulator Kinematics



Many slides adapted (with thanks!) from:
Siegwart, Nourbakhsh and Scaramuzza, Autonomous Mobile Robots
Renata Melamud, An Introduction to Robot Kinematics, CMU
Rick Parent, Computer Animation, Ohio State
Steve Rotenberg, Computer Animation, UCSD
Angela Sodemann, www.youtube.com/watch?v=lVjFhNv2N8o, ASU

Defining Frames for Joints

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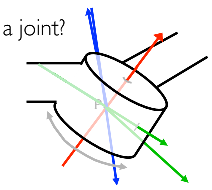
- What's the frame of reference for a joint?
  - Actually, completely flexible
- We usually choose:
  - 1 axis through the center of rotation/direction of displacement
  - 2 more perpendicular to that
    - Which can be any orientation!
- We can move the origin
  - P is no longer <0, 0, 0>
- To use DH method, choose frames carefully



Denavit-Hartenberg Method

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- Efficient method for finding transformation matrices

1. Set frames for all joints
   - This is actually the tricky part.
2. Calculate all DH parameters from frames
   - 4 DH parameters fully define position and orientation (not 6)
3. Populate DH parameter table
4. Populate joint-to-joint DH transformation matrices
   - Matrix for 0-1, matrix for 1-2, etc.
5. Multiply all matrices together, in order
   - 0-1 × 1-2 × 2-3 × …

Choosing Frames for DH

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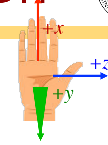
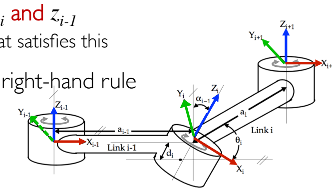


- *z* axis must be axis of motion
  - Rotation around *z* for revolute
  - Translation along *z* for prismatic
- x_i axis orthogonal to z_i and z_{i-1}
  - There's always a line that satisfies this
- *y* axis must follow the right-hand rule
  - Fingers point +*x*
  - Thumb points +*z*
  - Palm faces +*y*



- x_i axis must intersect z_{i-1} axis (may mean translating origin)

Find DH Parameters

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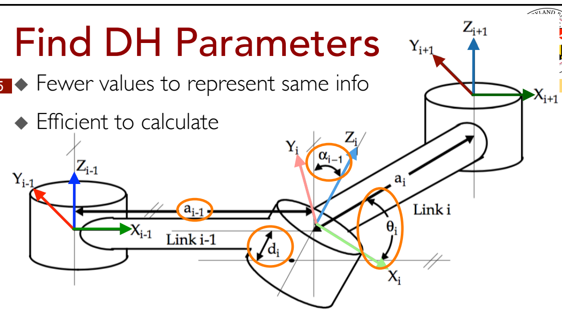
- Fewer values to represent same info
- Efficient to calculate



a_{i-1} : link length – distance Z_{i-1} and Z_i along X_i
α_{i-1} : link twist – angle Z_{i-1} and Z_i around X_i
d_i : link offset – distance X_{i-1} to X_i along Z_i
θ_i : joint angle – angle X_{i-1} and X_i around Z_i

Denavit-Hartenberg Method

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- A way of finding transformation matrix (quickly)

1. Assign DH frames to DoFs (previous slide)
   - This takes practice.
2. Create a parameter table
   - Rows = (# frames – 1)
   - Columns = 4 (always) ← your DH parameters θ, α, a, d

	θ	α	a	d
frame 0-1	θ_{0-1}	α_{0-1}	a_{0-1}	d_{0-1}
frame 1-2	θ_{1-2}	α_{1-2}	a_{1-2}	d_{1-2}
frame 2-3	…	…	…	…

Denavit-Hartenberg Method

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- Given parameter table,

3. Fill out transformation matrix* for each transition:

$$R_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_{i,i+1} & \sin\theta_i \sin\alpha_{i,i+1} & a_{i,i+1}\cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_{i,i+1} & -\cos\theta_i \sin\alpha_{i,i+1} & a_{i,i+1}\sin\theta_i \\ 0 & \sin\alpha_{i,i+1} & \cos\alpha_{i,i+1} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. And multiply. Ex: $R_2^0 = R_1^0 R_2^1$

- R_2^0 is the same matrix as would be found by other methods. DH is fast and efficient.

** If you'd like the derivation of this, I'll provide a link.*

Example: Rotation in Plane

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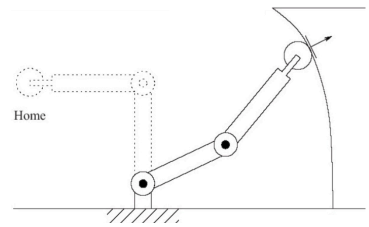

$x = a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2)$

$y = a_1 \sin\theta_1 + a_2 \sin(\theta_1 + \theta_2)$

$a_i =$ the length of ith link

Transformation i to i-1

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$\mathbf{a_{i-1}}$: distance $\mathbf{Z_{i-1}}$ and $\mathbf{Z_i}$ along $\mathbf{X_i}$
$\boldsymbol{\alpha_{i-1}}$: angle $\mathbf{Z_{i-1}}$ and $\mathbf{Z_i}$ around $\mathbf{X_i}$
} together: *screw displacement*

$$[X_i] = \mathrm{Trans}_{X_i}(a_{i,i+1})\,\mathrm{Rot}_{X_i}(\alpha_{i,i+1})$$

$\mathbf{d_i}$: distance $\mathbf{X_{i-1}}$ to $\mathbf{X_i}$ along $\mathbf{Z_i}$
$\boldsymbol{\theta_2}$: angle $\mathbf{X_{i-1}}$ and $\mathbf{X_i}$ around $\mathbf{Z_i}$
} together: *screw displacement*

$$[Z_i] = \mathrm{Trans}_{Z_i}(d_i)\,\mathrm{Rot}_{Z_i}(\theta_i)$$

- Coordinate transformation:

$${}^{i-1}T_i = [Z_i][X_i] = \mathrm{Trans}_{Z_i}(d_i)\,\mathrm{Rot}_{Z_i}(\theta_i)\,\mathrm{Trans}_{X_i}(a_{i,i+1})\,\mathrm{Rot}_{X_i}(\alpha_{i,i+1}),$$

Transformation i to i-1

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$$\mathrm{Trans}_{Z_i}(d_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathrm{Rot}_{Z_i}(\theta_i) = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{Trans}_{X_i}(a_{i,i+1}) = \begin{bmatrix} 1 & 0 & 0 & a_{i,i+1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathrm{Rot}_{X_i}(\alpha_{i,i+1}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_{i,i+1} & -\sin\alpha_{i,i+1} & 0 \\ 0 & \sin\alpha_{i,i+1} & \cos\alpha_{i,i+1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation in DH:

$${}^{i-1}T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_{i,i+1} & \sin\theta_i \sin\alpha_{i,i+1} & a_{i,i+1}\cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_{i,i+1} & -\cos\theta_i \sin\alpha_{i,i+1} & a_{i,i+1}\sin\theta_i \\ 0 & \sin\alpha_{i,i+1} & \cos\alpha_{i,i+1} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix},$$