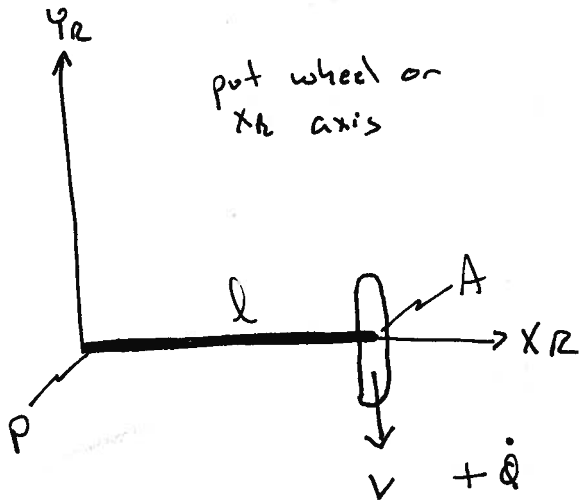
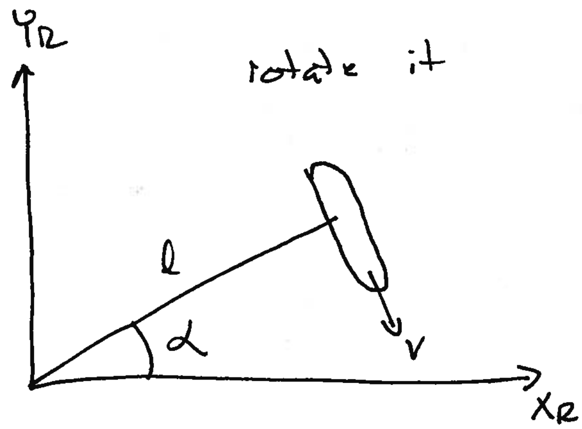
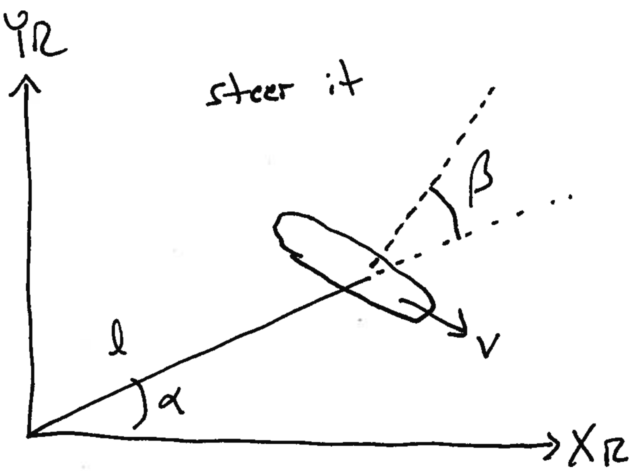
3/12/10

Reminder of setup

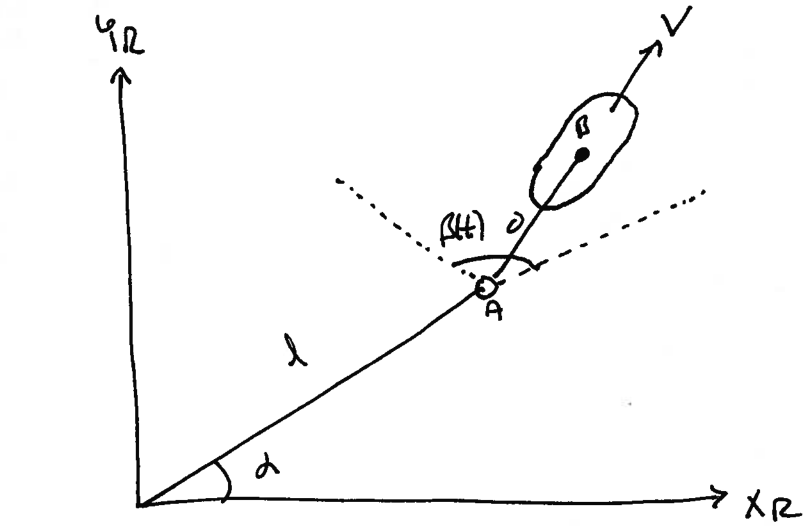
i)


ii)


iii)


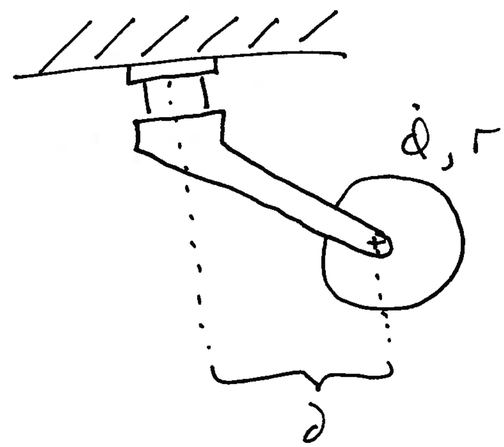
what is rolling constraint?
what is sliding constraint?

Castor wheel



$\alpha = 0$

Where is wheel when $\beta = 0$? Directly below A on X_R axis.



Contact point offset from axis of rotation by distance d.

Rolling constraint

$$\Big[\underbrace{\sin(\alpha+\beta)}_{=\cos(\alpha+\beta-\pi/2)} \quad -\cos(\alpha+\beta) \quad -l\cos\beta\Big]\, R(\theta)\, \dot{\xi}_I - r\dot{\varphi} = 0$$

Offset plays no role in motion aligned with wheel plane.

Sliding constraint

$$\Big[\cos(\alpha+\beta) \quad \sin(\alpha+\beta) \quad \overbrace{d+}^{\text{new!}}\, l\sin\beta\Big]\, R(\theta)\, \dot{\xi}_I + \overbrace{d\dot{\beta}}^{\text{new!}} = 0$$

motion ⊥ to wheel due to $\dot{x}_R$

motion ⊥ to wheel due to $\dot{y}_R$

motion ⊥ to wheel due to $\dot{\theta}$

why?

with no d offset

motion in robot's coordinate frame

what is this?

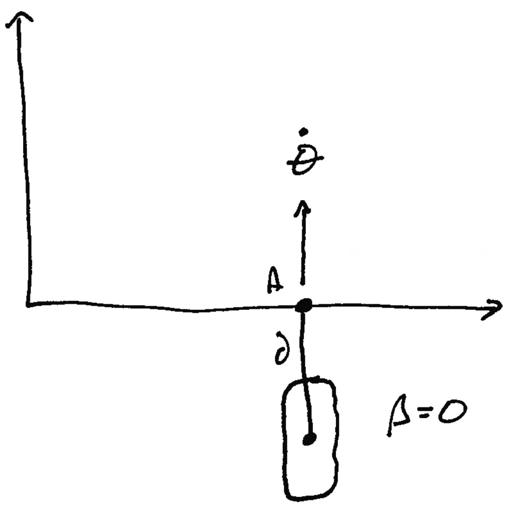
linear motion of wheel ⊥ to axis due to steering

What about $\partial + l \sin\beta$ term?
I have my doubts!!

First, for the instantaneous constraint you have to assume that β is fixed, so the $P \to A \to B$ linkage is a fixed structure.

What is distance of B from P? Call it m.
Then point B is moving with linear velocity $\dot{\theta} m$.

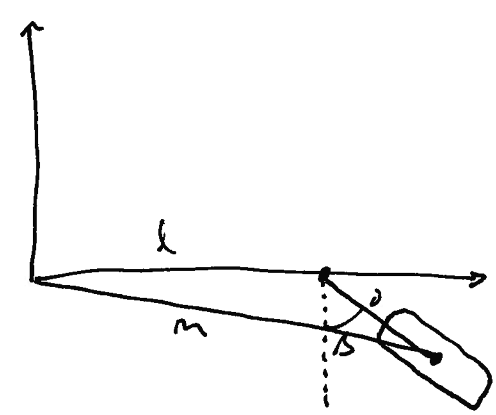
What component of that motion is orthogonal to wheel plane?

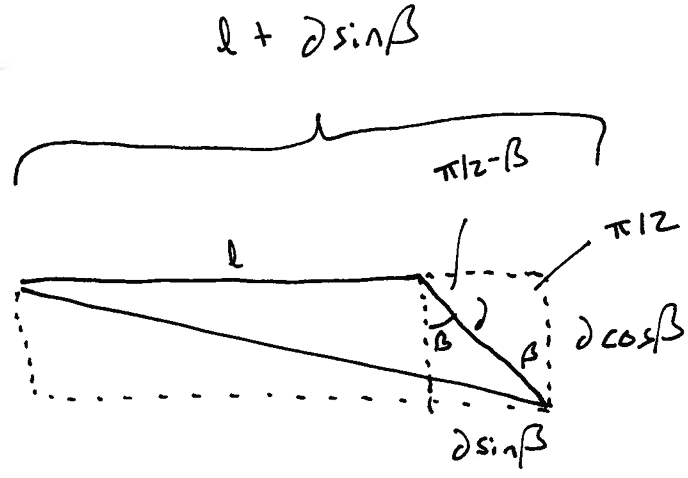


$\angle$ between $\dot{\theta}$ motion & $\perp$ wheel plane is $\frac{\pi}{2} - \beta$

$\cos\left(\frac{\pi}{2} - \beta\right) = \sin(\beta)$

What is m?





$$m^2 = (l + \partial \sin\beta)^2 + (\partial \cos\beta)^2$$

$$= l^2 + 2l\partial \sin\beta + \partial^2 \sin^2\beta + \partial^2 \cos^2\beta$$

$$= l^2 + 2l\partial \sin\beta + \partial^2(\sin^2\beta + \cos^2\beta)$$

$$= l^2 + 2l\partial \sin\beta + \partial^2$$

$$m = \sqrt{l^2 + 2l\partial \sin\beta + \partial^2}$$

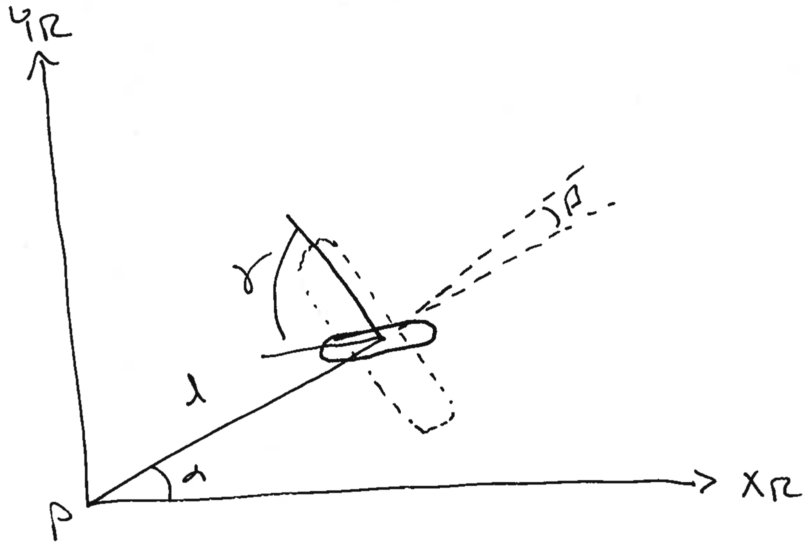
What do you think class??

what if $\beta = 0$?

what if $\beta = \pi/2$?

Why omnidirectional?

Swedish wheel



Cannot slide ⊥ to orientation of rollers so movement in that direction must be compensated for by rolling of the wheel

$$[\sin(\alpha+\beta+\gamma) \quad -\cos(\alpha+\beta+\gamma) \quad -l\cos(\beta+\gamma)]\, R(\theta)\, \xi_I - r\dot{\varphi}\cos\gamma = 0$$

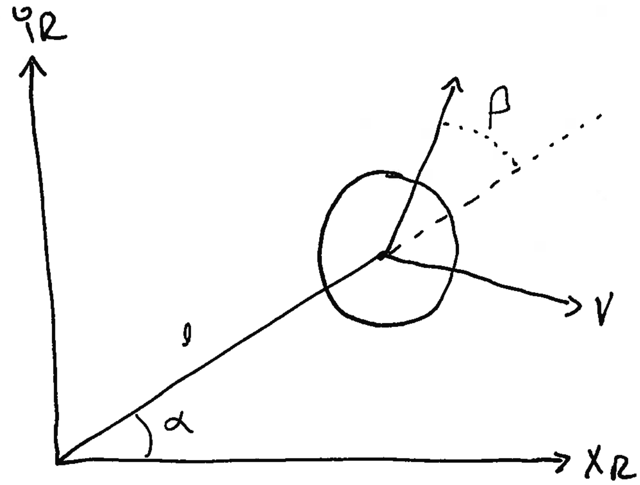
Orthogonal to that direction motion is not constrained due to free rotation of rollers

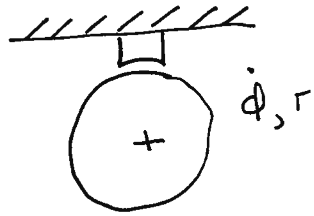
$$[\cos(\alpha+\beta+\gamma) \quad \sin(\alpha+\beta+\gamma) \quad l\sin(\beta+\gamma)]\, R(\theta)\, \dot{\xi}_I - r\dot{\varphi}\sin\gamma - r_{sw}\dot{\varphi}_{sw} = 0$$

what if $\gamma = 0$?
what if $\gamma = \frac{\pi}{2}$?

Why omnidirectional??

Spherical wheel





Rolling

$$[\sin(\alpha+\beta) \quad -\cos(\alpha+\beta) \quad -l\cos\beta]\, R(\theta)\, \dot{\xi}_I - r\dot{\phi} = 0$$

Sliding

$$[\cos(\alpha+\beta) \quad \sin(\alpha+\beta) \quad l\sin\beta]\, R(\theta)\, \dot{\xi}_I = 0$$

β is a <u>free</u> variable.

Can always choose β that satisfies $=0$ constraints.

Putting it all together

- fixed wheel
- steerable wheel
- castor
- swedish
- spherical

$\left.\begin{array}{l}\text{castor}\\ \text{swedish}\\ \text{spherical}\end{array}\right\}$ impose no constraints so we ignore them

$$\underbrace{N}_{} = \underset{\substack{\uparrow\\ \text{fixed wheels}}}{N_f} + \underset{\nwarrow \text{ steerable wheels}}{N_s}$$

$\beta_s(t)$ = steering angles of steerable wheels

β_f = " " " fixed wheels

$$\varphi(t) = \begin{bmatrix} \varphi_f(t) \\ \varphi_s(t) \end{bmatrix} = N \times 1 \text{ matrix of wheel rotational positions}$$

$\varphi_f(t) = N_f \times 1$

$\varphi_s(t) = N_s \times 1$

Rolling constraints of all wheels

$$J_1(\beta_s) \underbrace{R(\theta) \dot{\xi}_I}_{?} - J_2 \dot{\varphi} = 0$$

$J_2 = N \times N$ diagonal matrix of wheel radii

$J_1 = N \times 3$ matrix of terms in rolling constraint

$$J_1 = \begin{bmatrix} J_{1,f} \\ J_{1,s} \end{bmatrix}$$ fixed / steerable

Sliding constraint

$$C_1 \; R(\theta)\, \dot{\xi}_I = 0$$

↑ what do you think C_1 is?

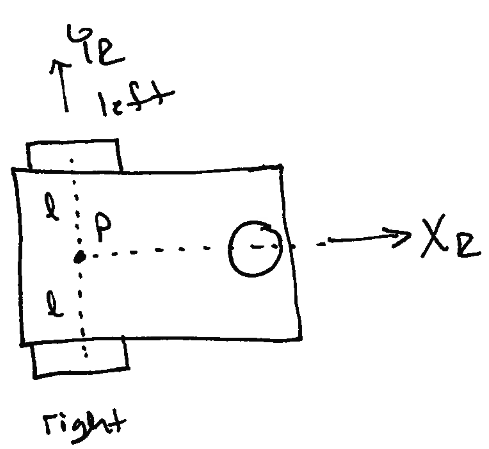
$$C_1 = \begin{bmatrix} C_{1f} \\ C_{1s} \end{bmatrix}$$ $N \times 3$ matrix of terms in sliding constraint

All constraints summarized in linear system

$$\begin{bmatrix} J_1 \\ C_1 \end{bmatrix} R(\theta)\, \dot{\xi}_I = \begin{bmatrix} J_2 \dot{\varphi} \\ 0 \end{bmatrix}$$

note error on pg. 63 in eq. 3.28, no dot on φ

Consider differential drive robot



what is α, β?

right, $\alpha = -\pi/2$

$\beta = \pi$ so forward motion is $\rightarrow X_R$

left $\alpha = \pi/2$

$\beta = 0$

J_1 for rolling

$\sin(\alpha+\beta) \qquad -\cos(\alpha+\beta) \qquad -l\cos\beta$

C_1 for sliding

$\cos(\alpha+\beta) \qquad \sin(\alpha+\beta) \qquad l\sin\beta$

$$\begin{array}{l} \text{right} \\ \text{left} \\ \\ \end{array} \begin{bmatrix} \begin{bmatrix} 1 & 0 & l \\ 1 & 0 & -l \end{bmatrix} \\ \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \end{bmatrix} R(\theta)\dot{\xi}_I = \begin{bmatrix} J_2\dot{\varphi} \\ 0 \end{bmatrix}$$

$$\dot{\xi}_I = R(\theta)^{-1} \begin{bmatrix} 1 & 0 & l \\ 1 & 0 & -l \\ 0 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} J_2\dot{\varphi} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_I \\ \dot{y}_I \\ \dot{\theta} \end{bmatrix} = R(\theta)^{-1} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \\ \frac{1}{2l} & -\frac{1}{2l} & 0 \end{bmatrix} \begin{bmatrix} J_2\dot{\varphi} \\ 0 \end{bmatrix}$$

So? For any desired $\dot{\xi}_I$ I can solve for $\dot{\varphi}$. Why? How? What does that mean?