Kinematics

kinematics - study of motion without consideration of the forces involved

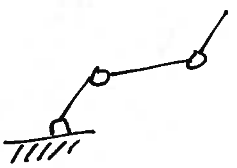
dynamics - study of the relationship between forces and the accelerations they produce

different question in mobile robotics related to veloci[ty]

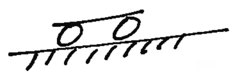
Fixed manipulator vs. mobile robot

forward kinematics: given joint ∠s where is end effector

inverse kinematics: given end effector location what are joint ∠s



what if 100lb weight here?



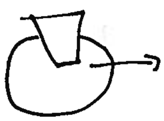
position of endpoint can be computed from instantaneous sensor values

Dynamics important's swing up task

position must be integrated over time

The path from here

i) understand how individual wheels do and do not move



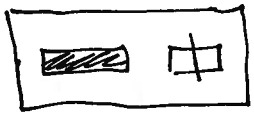
OK

not ok to skid laterally = constraint on motion

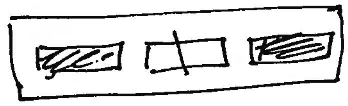
ii) framework for expressing motion in local & global reference frames

define

iii) derive forward kinematic models for robot based on its geometry & wheels to get constraints on motion of robot e.g.



what can this do?

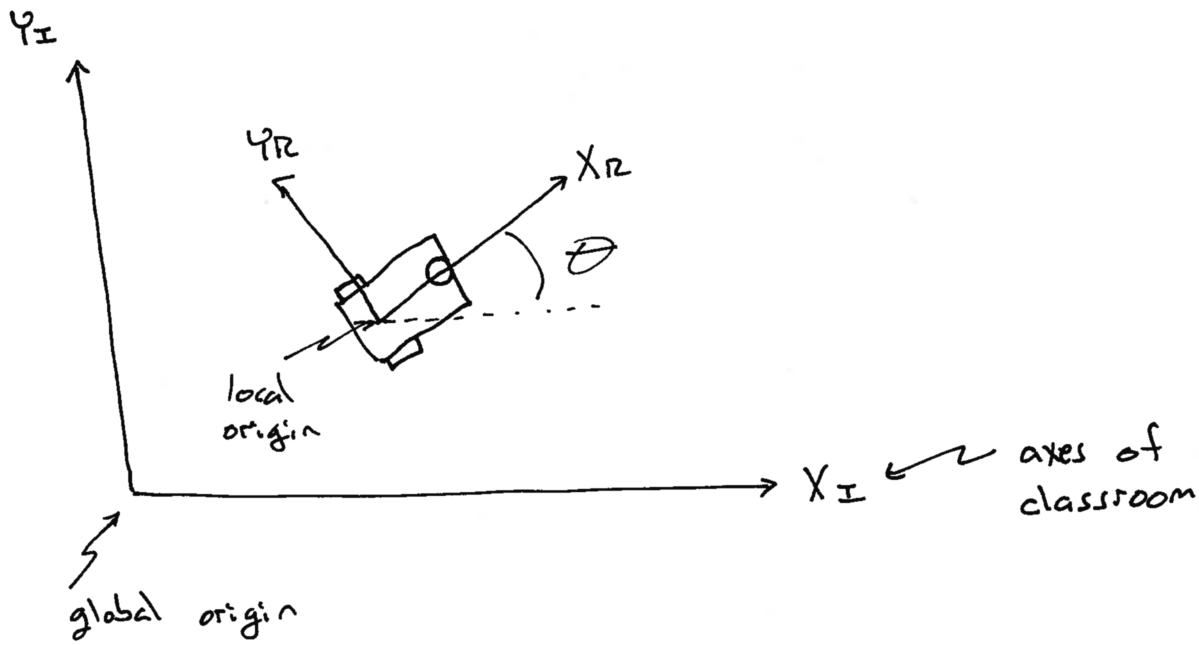


what can this do?

Why? We'll do that formally.

Intuition

To describe motion we need to first describle location in 2D horizontal floor plane.



position in global reference frame is

$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$ a vector or 3x1 matrix

what are the components?

"chi" = Xi

velocity in global reference frame is

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$ what are these components?

Likewise for $\dot{\xi}_R$. Note that ξ_R never changes because robot carries it around

We'd like to know how $\dot{\xi}_I$ and $\dot{\xi}_R$ are related. Why?

- path in global frame requires generating $\dot{\xi}_R$ that leads to desired $\dot{\xi}_I$

How do we translate between the two?

Let $R(\theta)$ be a rotation matrix s.t.

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then $\dot{\xi}_R = R(\theta)\,\dot{\xi}_I$

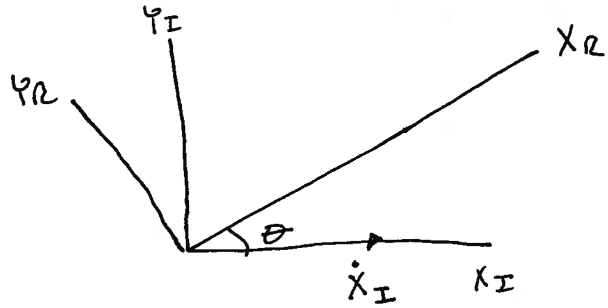
$$= \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_I \\ \dot{y}_I \\ \dot{\theta}_I \end{bmatrix}$$

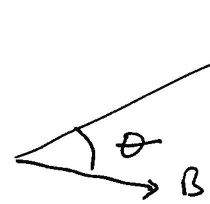
$$= \begin{bmatrix} \dot{x}_I \cos\theta + \dot{y}_I \sin\theta \\ -\dot{x}_I \sin\theta + \dot{y}_I \cos\theta \\ \theta_I \end{bmatrix} = \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{bmatrix}$$

Why does this make sense?

i) Why are there no sin/cos terms on θ_I?
Rate of rotation in global frame is same in local frame.

ii) $\dot{x}_R = \dot{x}_I \cos\theta + \dot{y}_I \sin\theta$





magnitude of A in dir. of B = $A\cos\theta$, likewise for $B\cos\theta$ in direction of A

Magnitude of $\dot{x}_I$ in direction of X_R is $\dot{x}_I \cos\theta$

Ask for angles

Easy to check correctness by trying $\theta = 0$ & $\theta = \frac{\pi}{2}$ why?

What is angle between y_I and X_R? $\pi/2 - \theta$

Projection is $\dot{y}_I \cos(\pi/2 - \theta) = \dot{y}_I \sin\theta$

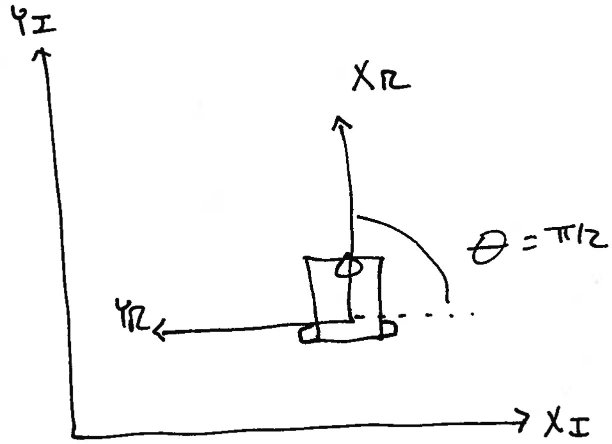
we add those two components to get result

iii) $\dot{y}_R = -\dot{x}_I \sin\theta + \dot{y}_I \cos\theta$

$\angle$ between X_I and y_R = $\theta + \pi/2$

$\angle$ between y_I and y_R = θ

so because $\cos(\theta + \pi/2) = -\sin\theta$ the result holds



Do this as a brief exercise in class

$$\dot{\xi}_R = R(\pi/2)\,\dot{\xi}_I$$

$$= \begin{bmatrix} \cos\frac{\pi}{2} & \sin\frac{\pi}{2} & 0 \\ -\sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_I \\ \dot{y}_I \\ \dot{\theta} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_I \\ \dot{y}_I \\ \dot{\theta} \end{bmatrix}$$

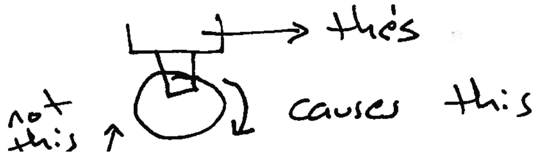
$$= \begin{bmatrix} \dot{y}_I \\ -\dot{x}_I \\ \dot{\theta} \end{bmatrix}$$

Why does this make sense?

Wheel kinematic constraints

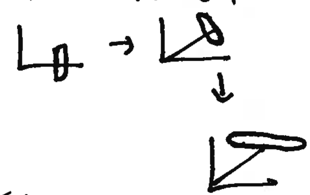
Express constraints on motions of individual wheels
Add them together to get constraints on entire robot

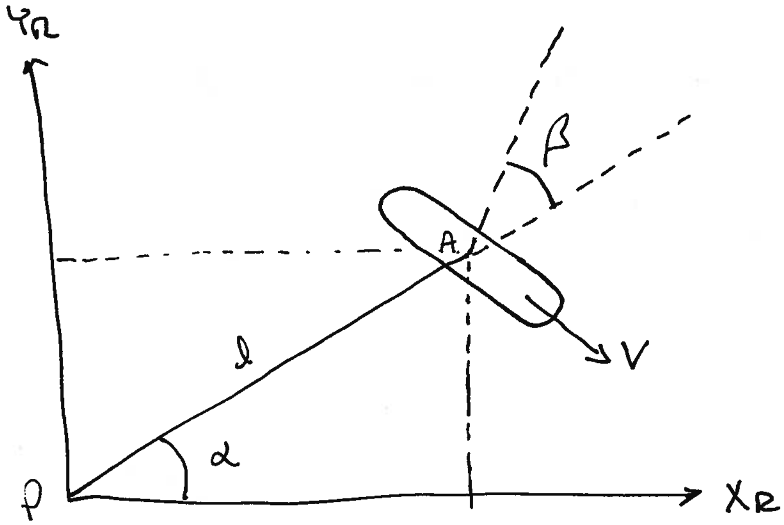
Assumptions
- plane of wheel remains veritcal
- single point of contact between wheel + ground
- no sliding at the point of contact

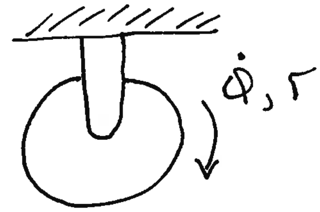


Fixed standard wheel

Do this and then shift wheel as I go. Have both in ∥ on board.







If robot moves globally like $\dot{\xi}_I$, what happens with wheel & what $\dot{\xi}_I$ are possible?

why ==?

Rolling constraint

$$\underbrace{[\sin(\alpha+\beta) \quad -\cos(\alpha+\beta) \quad (-l)\cos\beta]\, R(\theta)\, \dot{\xi}_I}_{} = r\dot{\phi}$$

$$\dot{\xi}_R = \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta} \end{bmatrix}$$

fact
$$\boxed{\cos(x - \tfrac{\pi}{2}) = \sin(x)}$$

result is a real number projection of $\dot{\xi}_R$ along V

∠ between X_R and V is ... $-(\alpha+\beta - \frac{\pi}{2})$ → $\cos[-(\alpha+\beta-\frac{\pi}{2})] = \sin(\alpha+\beta)$

∠ between Y_R and V is ... ~~$(\alpha+\beta) \to \cos$~~

$-(\alpha+\beta-\pi) \Rightarrow \cos[-(\alpha+\beta-\pi)] = -\cos(\alpha+\beta)$

A is moving ⊥ to P-A with velocity $l\dot{\theta}$

∠ between that vector and V is $\pi - \beta \to \cos(\pi-\beta) = -\cos(\beta)$

$\to -l\dot{\theta}\cos\beta$

Note this constraint can always be satisfied.

Sliding constraint

motion $\perp$ to v is zero

$$[a \quad b \quad c]\, R(\theta)\, \dot{\xi}_I = 0$$

what are a, b, c?

can this constraint always be satisfied? No!

$$\left.\begin{aligned} a &= \cos(\alpha+\beta) \\ b &= \sin(\alpha+\beta) \\ c &= l\sin\beta \end{aligned}\right\}$$ <u>ask students to do this exercise for 5 mins</u>

Example: $\alpha = \beta = 0$, $\theta = 0$ What does this look like?

Sliding constraint?

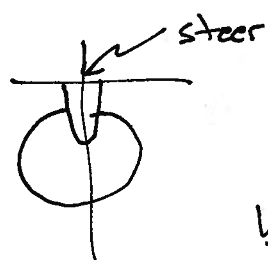
$$[1 \quad 0 \quad 0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_I \\ \dot{y}_I \\ \dot{\theta}_I \end{bmatrix} = 0$$

$$[1 \quad 0 \quad 0] \begin{bmatrix} \dot{x}_I \\ \dot{y}_I \\ \dot{\theta} \end{bmatrix} = 0 \quad \Rightarrow \dot{x}_I = 0$$

Does this make sense?

Steered standard wheel

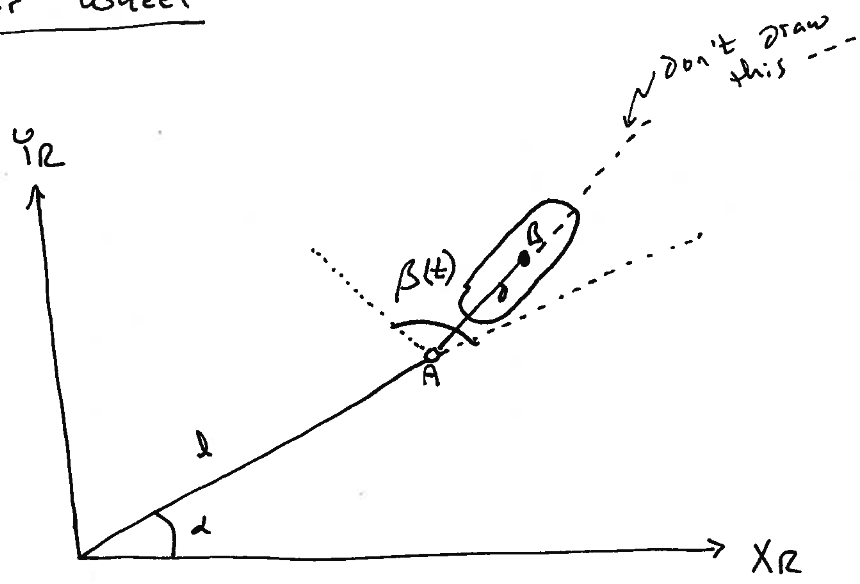
$\beta = \beta(t)$

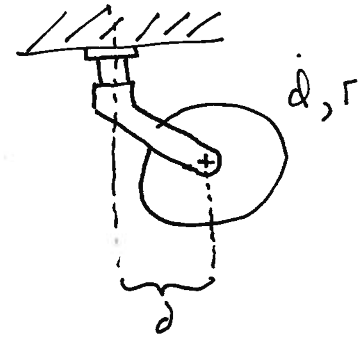


but imparts no forces, so equations are the same

but changing β affects motion over time

Castor wheel





wheel is offset from axis of rotation by ~~[illegible]~~ d.
Can be at steering $\angle$ $\beta(t)$

<u>Rolling</u> <u>constraint</u> is the same as for fixed/steered wheels because offset plays no role in motion aligned with wheel axis

$$[\sin(\alpha+\beta) \quad -\cos(\alpha+\beta) \quad -l\cos\beta] - r\dot{\varphi} = 0$$

<u>Sliding</u> <u>constraint</u>

$$[\cos(\alpha+\beta) \quad \sin(\alpha+\beta) \quad \underbrace{d}_{\text{new}} + l\sin\beta]\, R(\theta)\,\dot{\xi}_I + \underbrace{d\dot{\beta}}_{\text{new}} = 0$$

motion $\perp$ to wheel due to $\dot{X}_R$

motion $\perp$ to wheel due to $\dot{Y}_R$

I'm convinced this is wrong.
Imagine $\alpha = \beta = 0$
Does formula make sense?

explain this

In class activity. $\longrightarrow$