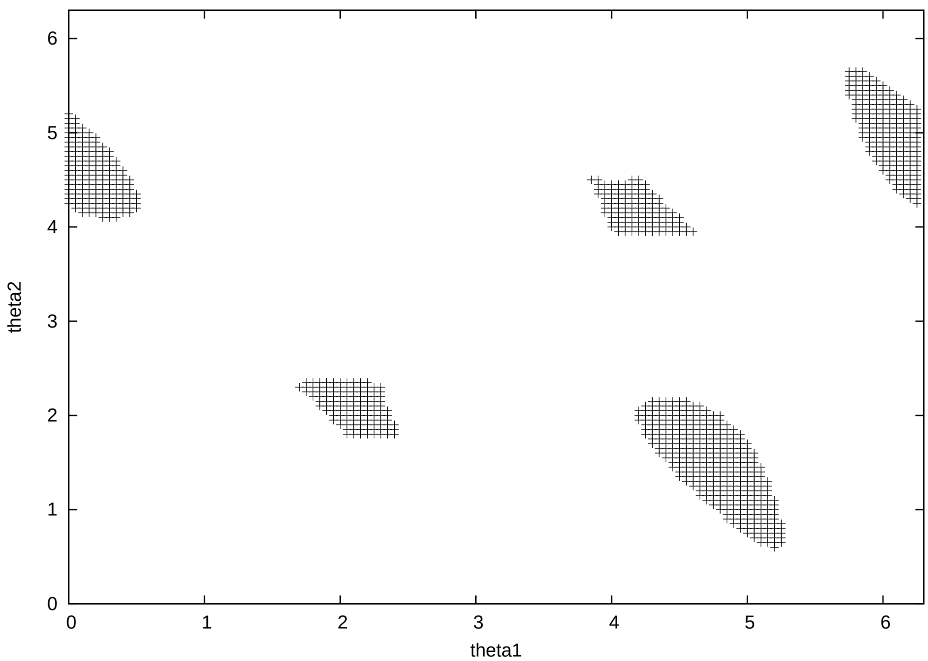
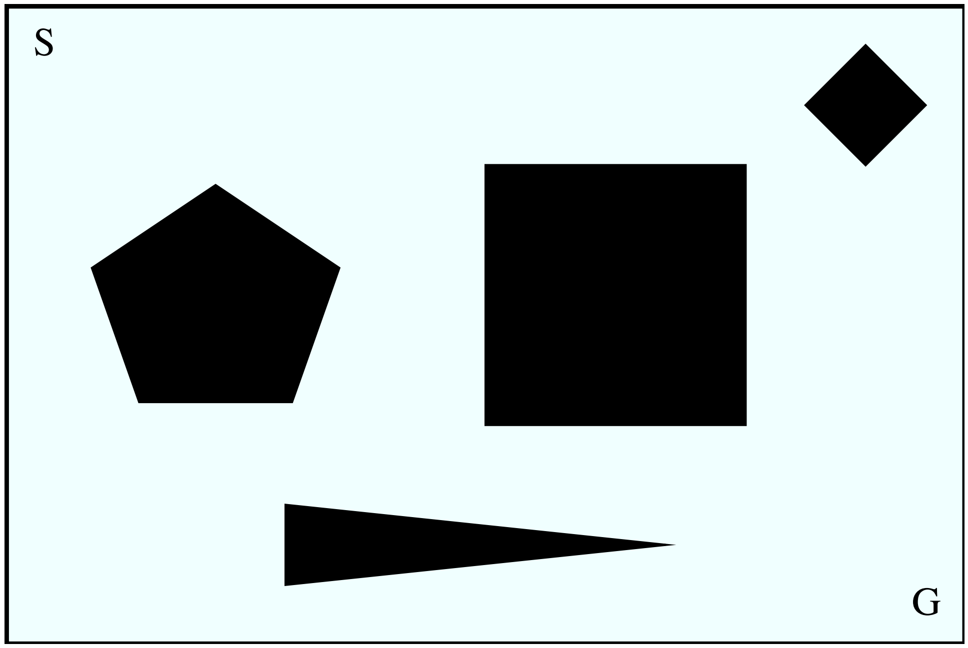
Introduction to Robotics - CMSC 479/679
Homework #4
Due Wednesday, May 9^{th} at the start of class

Kalman Filter: Suppose a *stationary* mobile robot facing a wall μ feet away takes repeated measurements of the distance to that wall. The sensor's value is normally distributed with mean μ and standard deviation σ. If the robot uses the Kalman filter to estimate its distance to the wall, folding in new measurements from the sensor as they are produced, what happens to the variance of the robot's estimate of its position? Why?

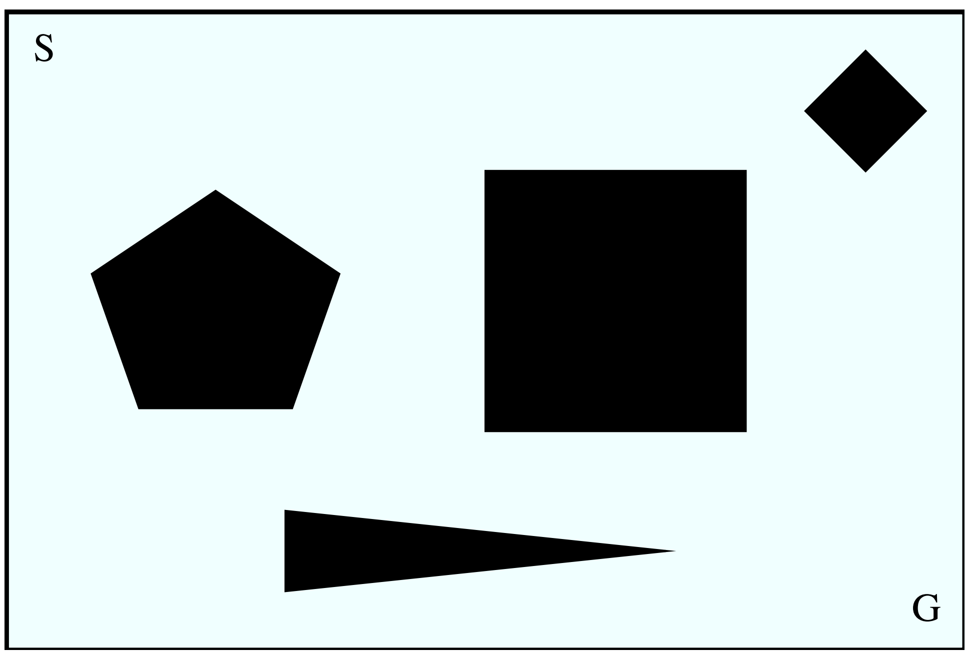
Configuration space: Consider the 2-dof robot arm shown on page 260 of the textbook. Assume that each link in the arm has length 1. Suppose the base of this arm is at location (0, 0) in *cartesian space*. Below is a map in the robot's *configuration space* where each dark point corresponds to a configuration in which the tip of the arm is somewhere inside an obstacle. (To draw this map, we assume that the obstacles can be penetrated by the arm. That is, the tip of the arm, and the remainder of the arm, can be inside an obstacle. This is clearly not realistic, but it is irrelevant to the point of this question.) Draw the obstacles in *cartesian space*. That is, what obstacles would give rise to the configuration space below? Hint: there are two obstacles.



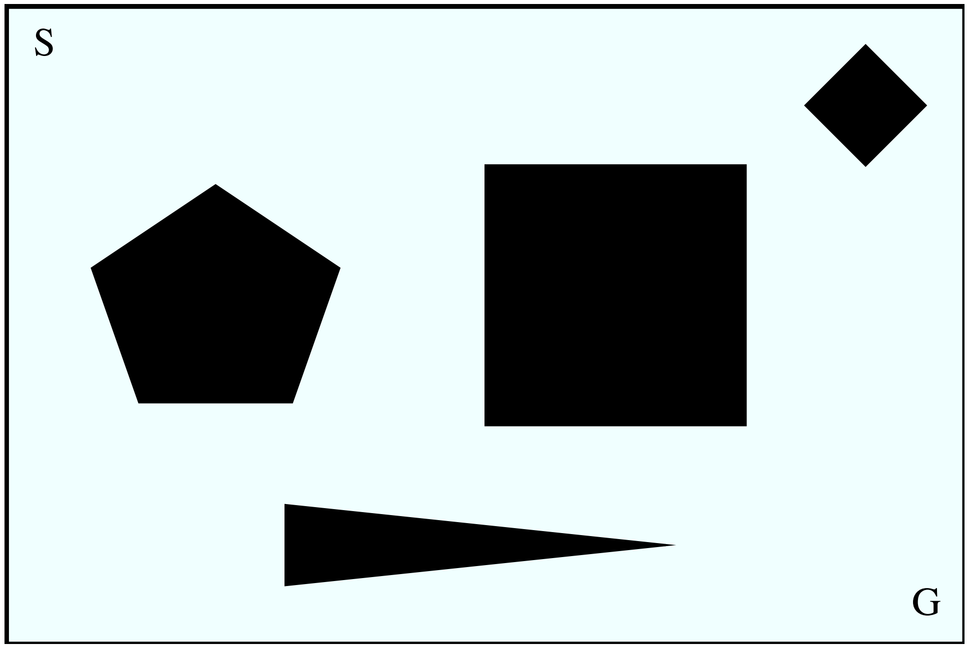
Visibility graph: Given the start (S) and goal (G) locations for a mobile robot as shown below, draw the corresponding visibility graph. Also show (perhaps in bold or in a different color) the shortest path from S to G.



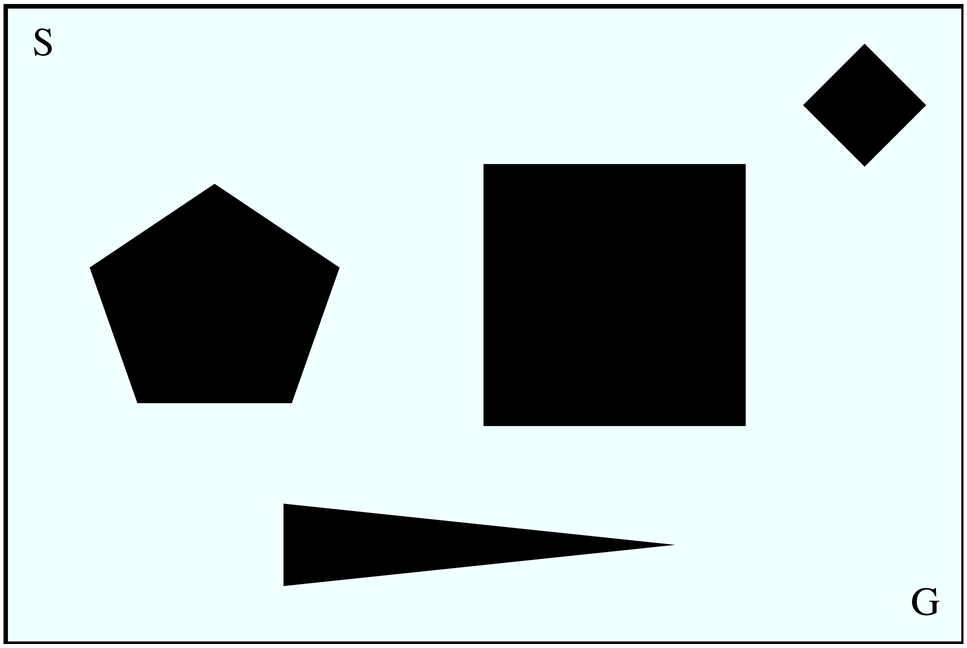
Voronoi diagram: Given the start (S) and goal (G) locations for a mobile robot as shown below, draw the corresponding Voronoi diagram. Also show (perhaps in bold or in a different color) the shortest path from S to G.



Exact cell decomposition: Given the start (S) and goal (G) locations for a mobile robot as shown below, draw the corresponding exact cell decomposition using vertical cell boundaries. Given the sequence of cell numbers that will be traversed along the shortest path from S to G.



Quad-tree decomposition: Given the start (S) and goal (G) locations for a mobile robot as shown below, draw the corresponding quad-tree decomposition (the book refers to this as *adaptive decomposition* in chapter 5). I'll leave it up to you to decide when the decomposition is fine-grained enough.



Potential fields: Suppose a robot is sitting still in a hallway 4 meters wide. The robot is facing one of the walls, it doesn't matter which one, and can only move forward and backward, it cannot turn. The left wall is at location $x = 0m$ and the right wall as at location $x = 4m$. Initially, the robot is 1 meter from the right-hand wall, at location $x = 3m$, and it's motion is controlled by the potential field method. The walls generate a repulsing potential as follows:

$$U(x_{robot}, x_{wall}) = 10|x_{robot} - x_{wall}|^{-2}$$

The robot weighs $2kg$ and can send a new motion command once a second to its drive, sending the first command when it is turned on at time $t = 0s$. Assume the robot's drive system has no upper bound on acceleration or change in acceleration (use Newton's law, $F = ma$, to compute acceleration).

- What will happen to the robot in this situation?
- Compute (by hand or by writing a small program) the robot's location at $t = 1s$, $t = 2s$, and $t = 3s$. Given initial velocity v_0 and constant acceleration a for t time steps, the final velocity is given by $v_0 + at$.