CMSC 641 - Fourier Practice Solutions

```
x1 = var('x1')
x2 = var('x2')
x3 = var('x3')
```

Practice Problem #1

Express the Sel function as sums of indicator functions as in previous exercises:

```
(1+x1)*(1+x2)*(1+x3)/8 - (1+x1)*(1+x2)*(1-x3)/8 + (1+x1)*(1-x2)*
(1+x3)/8 - (1+x1)*(1-x2)*(1-x3)/8 + (1-x1)*(1+x2)*(1+x3)/8 + (1-x1)*
(1+x2)*(1-x3)/8 - (1-x1)*(1-x2)*(1+x3)/8 - (1-x1)*(1-x2)*(1-x3)/8
```

```
1/8*(x1 + 1)*(x2 + 1)*(x3 + 1) - 1/8*(x1 - 1)*(x2 + 1)*(x3 + 1) -
1/8*(x1 + 1)*(x2 - 1)*(x3 + 1) - 1/8*(x1 - 1)*(x2 - 1)*(x3 + 1) +
1/8*(x1 + 1)*(x2 + 1)*(x3 - 1) + 1/8*(x1 - 1)*(x2 + 1)*(x3 - 1) -
1/8*(x1 + 1)*(x2 - 1)*(x3 - 1) + 1/8*(x1 - 1)*(x2 - 1)*(x3 - 1)
```

Now expand the expression and simplify:

```
expand( (1+x1)*(1+x2)*(1+x3)/8 - (1+x1)*(1+x2)*(1-x3)/8 + (1+x1)*(1-
x2)*(1+x3)/8 - (1+x1)*(1-x2)*(1-x3)/8 + (1-x1)*(1+x2)*(1+x3)/8 + (1-
x1)*(1+x2)*(1-x3)/8 - (1-x1)*(1-x2)*(1+x3)/8 - (1-x1)*(1-x2)*(1-x3)/8
)
```

```
-1/2*x1*x2 + 1/2*x1*x3 + 1/2*x2 + 1/2*x3
```

Practice Problem #2

Express the $Sort_4$ function as sums of indicator functions as in previous exercises. Here I have written the positive terms first, followed by the negative terms.

```
x4 = var('x4')
(1+x1)*(1+x2)*(1-x3)*(1+x4)/16 + (1+x1)*(1-x2)*(1+x3)*(1+x4)/16 +
(1+x1)*(1-x2)*(1-x3)*(1+x4)/16 + (1-x1)*(1+x2)*(1-x3)*(1+x4)/16 + (1-
x1)*(1+x2)*(1+x3)*(1-x4)/16 + (1+x1)*(1-x2)*(1+x3)*(1-x4)/16 + (1-x1)*
(1-x2)*(1+x3)*(1-x4)/16 + (1-x1)*(1+x2)*(1-x3)*(1-x4)/16 - (1+x1)*
(1+x2)*(1+x3)*(1+x4)/16 - (1-x1)*(1+x2)*(1+x3)*(1+x4)/16 - (1-x1)*(1-
x2)*(1+x3)*(1+x4)/16 - (1-x1)*(1-x2)*(1-x3)*(1+x4)/16 - (1-x1)*(1-x2)*
(1-x3)*(1-x4)/16 - (1+x1)*(1-x2)*(1-x3)*(1-x4)/16 - (1+x1)*(1+x2)*(1-
x3)*(1-x4)/16 - (1+x1)*(1+x2)*(1+x3)*(1-x4)/16
```

```
-1/16*(x1 + 1)*(x2 + 1)*(x3 + 1)*(x4 + 1) + 1/16*(x1 - 1)*(x2 +
1)*(x3 + 1)*(x4 + 1) - 1/16*(x1 + 1)*(x2 - 1)*(x3 + 1)*(x4 + 1) -
```

```
1/16*(x1 - 1)*(x2 - 1)*(x3 + 1)*(x4 + 1) - 1/16*(x1 + 1)*(x2 +
1)*(x3 - 1)*(x4 + 1) + 1/16*(x1 - 1)*(x2 + 1)*(x3 - 1)*(x4 + 1) +
1/16*(x1 + 1)*(x2 - 1)*(x3 - 1)*(x4 + 1) + 1/16*(x1 - 1)*(x2 -
1)*(x3 - 1)*(x4 + 1) + 1/16*(x1 + 1)*(x2 + 1)*(x3 + 1)*(x4 - 1) +
1/16*(x1 - 1)*(x2 + 1)*(x3 + 1)*(x4 - 1) + 1/16*(x1 + 1)*(x2 -
1)*(x3 + 1)*(x4 - 1) - 1/16*(x1 - 1)*(x2 - 1)*(x3 + 1)*(x4 - 1) -
1/16*(x1 + 1)*(x2 + 1)*(x3 - 1)*(x4 - 1) - 1/16*(x1 - 1)*(x2 +
1)*(x3 - 1)*(x4 - 1) + 1/16*(x1 + 1)*(x2 - 1)*(x3 - 1)*(x4 - 1) -
1/16*(x1 - 1)*(x2 - 1)*(x3 - 1)*(x4 - 1)
```

Now expand the expression and simplify:

```
expand( (1+x1)*(1+x2)*(1-x3)*(1+x4)/16 + (1+x1)*(1-x2)*(1+x3)*
(1+x4)/16 + (1+x1)*(1-x2)*(1-x3)*(1+x4)/16 + (1-x1)*(1+x2)*(1-x3)*
(1+x4)/16 + (1-x1)*(1+x2)*(1+x3)*(1-x4)/16 + (1+x1)*(1-x2)*(1+x3)*(1-
x4)/16 + (1-x1)*(1-x2)*(1+x3)*(1-x4)/16 + (1-x1)*(1+x2)*(1-x3)*(1-
x4)/16 - (1+x1)*(1+x2)*(1+x3)*(1+x4)/16 - (1-x1)*(1+x2)*(1+x3)*
(1+x4)/16 - (1-x1)*(1-x2)*(1+x3)*(1+x4)/16 - (1-x1)*(1-x2)*(1-x3)*
(1+x4)/16 - (1-x1)*(1-x2)*(1-x3)*(1-x4)/16 - (1+x1)*(1-x2)*(1-x3)*(1-
x4)/16 - (1+x1)*(1+x2)*(1-x3)*(1-x4)/16 - (1+x1)*(1+x2)*(1+x3)*(1-
x4)/16 )
```

```
-1/2*x1*x2 - 1/2*x2*x3 + 1/2*x1*x4 - 1/2*x3*x4
```