Chapter 3

(b) Semantics

Semantics Overview

• Syntax is about “form” and semantics about “meaning”.
• The boundary between syntax and semantics is not always clear.
• First we’ll look at issues close to the syntax end, what Sebesta calls “static semantics”, and the technique of attribute grammars.
• Then we’ll sketch three approaches to defining “deeper” semantics
  (1) Operational semantics
  (2) Axiomatic semantics
  (3) Denotational semantics

Motivation

• Capturing what a program in some programming language means is very difficult
• We can’t really do it in any practical sense for most work-a-day programming languages (e.g., C, C++, Java, Perl).
• Q: Why is worth trying?
• A: Program verification.
• Program Verification: the process of formal proving, that the computer program does exactly what is stated in the program specification it was written to realize.
  http://www.wikipedia.org/wiki/Program_verification

Program Verification

• Program verification can be done for simple programming languages
• It requires a formal specification for what the program should do – e.g., what it’s inputs will be and what actions it will take or output it will generate given the inputs
• That’s a hard task in itself!
• There are applications where it is worth it to (1) use a simplified programming language, (2) work out formal specs for a program, (3) capture the semantics of the simplified PL and (4) do the hard work of putting it all together and proving program correctness.
• What are they?
Program Verification

• There are applications where it is worth it to (1) use a simplified programming language, (2) work out formal specs for a program, (3) capture the semantics of the simplified PL and (4) do the hard work of putting it all together and proving program correctness. Like…
  • Security and encryption
  • Financial transactions
  • Applications on which lives depend (e.g., healthcare, aviation)
  • Expensive, one-shot, unreparable applications (Martian rover,)
  • Hardware design (e.g. Pentium chip)

Ariane 5

• It took the European Space Agency 10 years and $7 billion to produce Ariane 5, a giant rocket capable of hurling a pair of three-ton satellites into orbit with each launch and intended to give Europe overwhelming supremacy in the commercial space business.
• All it took to explode that rocket less than a minute into its maiden voyage in June 1996, scattering fiery rubble across the mangrove swamps of French Guiana, was a small computer program trying to stuff a 64-bit number into a 16-bit space.

Intel Pentium Bug

• In the mid 90’s a bug was found in the floating point hardware in Intel’s latest Pentium microprocessor.
• Unfortunately, the bug was only found after many had been made and sold.
• The bug was subtle, effecting only the 9th decimal place of some computations.
• But users cared.
• Intel had to recall the chips and took a $500M write-off

So…

• While automatic program verification is a long range goal
• which might be restricted to applications where the extra cost is justified
• we should try to design programming languages that help, rather than hinder, our ability to make progress in this area.
• we should continue research on the semantics of programming languages
• And the ability to prove program correctness
Semantics

- First we’ll look at issues close to the syntax end, what Sebesta calls “static semantics”, and the technique of attribute grammars.
- Then we’ll sketch three approaches to defining “deeper” semantics
  1. Operational semantics
  2. Axiomatic semantics
  3. Denotational semantics

Static Semantics

Static semantics covers some language features that are difficult or impossible to handle in a BNF/CFG.

It is also a mechanism for building a parser which produces a “abstract syntax tree” of its input.

Categories attribute grammars can handle:
  - Context-free but cumbersome (e.g. type checking)
  - Noncontext-free (e.g. variables must be declared before they are used)

Attribute Grammars

Attribute Grammars (AGs) (Knuth, 1968)
- CFGs cannot describe all of the syntax of programming languages
- Additions to CFGs to carry some “semantic” info along through parse trees

Primary value of AGs:
- Static semantics specification
- Compiler design (static semantics checking)

Attribute Grammar Example

In Ada we have the following rule to describe procedure definitions:

```
<proc> -> procedure <procName> <procBody> end <procName> ;
```

But, of course, the name after “procedure” has to be the same as the name after “end”.

This is not possible to capture in a CFG (in practice) because there are too many names.

Solution: associate simple attributes with nodes in the parse tree and add a “semantic” rules or constraints to the syntactic rule in the grammar.

```
<procName>[1].string = <procName>[2].string
```
**Attribute Grammars**

Def: An *attribute grammar* is a CFG $G=(S,N,T,P)$ with the following additions:
- For each grammar symbol $x$ there is a set $A(x)$ of attribute values.
- Each rule has a set of functions that define certain attributes of the nonterminals in the rule.
- Each rule has a (possibly empty) set of predicates to check for attribute consistency.

A Grammar is formally defined by specifying four components.
- $S$ is the start symbol
- $N$ is a set of non-terminal symbols
- $T$ is a set of terminal symbols
- $P$ is a set of productions or rules

Let $X_0 \rightarrow X_1 \ldots X_n$ be a rule.

Functions of the form $S(X_0) = f(A(X_1), \ldots, A(X_n))$ define *synthesized attributes*.

Functions of the form $I(X_i) = f(A(X_0), \ldots, A(X_n))$ for $i \leq j \leq n$ define *inherited attributes*.

Initially, there are *intrinsic attributes* on the leaves.

**Example:** expressions of the form $\text{id} + \text{id}$
- $\text{id}$'s can be either int_type or real_type
- types of the two $\text{id}$'s must be the same
- type of the expression must match its expected type

**BNF:**
\[
<\text{expr}> \rightarrow <\text{var}> + <\text{var}>
\]
\[
<\text{var}> \rightarrow \text{id}
\]

**Attributes:**
- actual_type - synthesized for $<\text{var}>$ and $<\text{expr}>
- expected_type - inherited for $<\text{expr}>$
Attribute Grammars

Attribute Grammar:

   Semantic rules:
   `<expr>.actual_type ← <var>[1].actual_type`
   Predicate:
   `<var>[1].actual_type = <var>[2].actual_type`
   `<expr>.expected_type = <expr>.actual_type`

2. Syntax rule: `<var> -> id`
   Semantic rule:
   `<var>.actual_type ← lookup (id, <var>)`

Attribute Grammars (continued)

How are attribute values computed?

• If all attributes were inherited, the tree could be decorated in top-down order.
• If all attributes were synthesized, the tree could be decorated in bottom-up order.
• In many cases, both kinds of attributes are used, and it is some combination of top-down and bottom-up that must be used.

Dynamic Semantics

No single widely acceptable notation or formalism for describing semantics.

The general approach to defining the semantics of any language L is to specify a general mechanism to translate any sentence in L into a set of sentences in another language or system that we take to be well defined.

Here are three approaches we’ll briefly look at:
  – Operational semantics
  – Axiomatic semantics
  – Denotational semantics
Operational Semantics

• Idea: describe the meaning of a program in language L by specifying how statements effect the state of a machine, (simulated or actual) when executed.

• The change in the state of the machine (memory, registers, stack, heap, etc.) defines the meaning of the statement.

• Similar in spirit to the notion of a Turing Machine and also used informally to explain higher-level constructs in terms of simpler ones, as in:

```
c statement     operational semantics
for(e1;e2;e3)   e1;
{<body>}        loop: if e2=0 goto exit
                 <body> e3;
                 goto loop
exit:
```

Operational Semantics

• To use operational semantics for a high-level language, a virtual machine is needed

• A hardware pure interpreter would be too expensive

• A software pure interpreter also has problems:
  • The detailed characteristics of the particular computer would make actions difficult to understand
  • Such a semantic definition would be machine-dependent

A better alternative: A complete computer simulation

• Build a translator (translates source code to the machine code of an idealized computer)

• Build a simulator for the idealized computer

Evaluation of operational semantics:

• Good if used informally

• Extremely complex if used formally (e.g. VDL)

Vienna Definition Language

• VDL was a language developed at IBM Vienna Labs as a language for formal, algebraic definition via operational semantics.

• It was used to specify the semantics of PL/I.

• See: The Vienna Definition Language, P. Wegner, ACM Comp Surveys 4(1):5-63 (Mar 1972)

• The VDL specification of PL/I was very large, very complicated, a remarkable technical accomplishment, and of little practical use.
Axiomatic Semantics

• Based on formal logic (first order predicate calculus)
• Original purpose: formal program verification
• Approach: Define axioms and inference rules in logic for each statement type in the language (to allow transformations of expressions to other expressions)
• The expressions are called assertions and are either
  • Preconditions: An assertion before a statement states the relationships and constraints among variables that are true at that point in execution
  • Postconditions: An assertion following a statement

Logic 101

Propositional logic:
Logical constants: true, false
Propositional symbols: P, Q, S, ... that are either true or false
Logical connectives: ∧ (and), ∨ (or), ⇒ (implies), ⇔ (is equivalent), ¬ (not) which are defined by the truth tables below.
Sentences are formed by combining propositional symbols, connectives and parentheses and are either true or false. e.g.: P ∧ Q ⇔ ¬(¬P ∨ ¬Q)

First order logic adds
(1) Variables which can range over objects in the domain of discourse
(2) Quantifiers including: ∀ (forall) and ∃ (there exists)
(3) Predicates to capture domain classes and relations
Examples: (∀p) (∀q) p ∧ q ⇔ ¬(¬p ∨ ¬q)
∀x prime(x) ⇒ ∃y prime(y) ∧ y > x

Axiomatic Semantics

• A weakest precondition is the least restrictive precondition that will guarantee the postcondition

Notation:
{P} Statement {Q}

precondition postcondition

Example:
{?} a := b + 1 {a > 1}
We often need to infer what the precondition must be for a given postcondition
One possible precondition: {b > 10}
Weakest precondition: {b > 0}
Example: Assignment Statements

Here’s how we might define a simple assignment statement of the form \( x := e \) in a programming language.

- \( \{ Q_{x := E} \} \ x := E \ \{ Q \} \)
- Where \( Q_{x := E} \) means the result of replacing all occurrences of \( x \) with \( E \) in \( Q \)

So from

\( \{ Q \} \ a := b/2-1 \ \{ a < 10 \} \)

We can infer that the weakest precondition \( Q \) is

\( b/2-1 < 10 \) or \( b < 22 \)

Axiomatic Semantics

- **The Rule of Consequence:**
  \[
  \{ P \} \ S \ \{ Q \}, \quad P' \Rightarrow P, \quad Q' \Rightarrow Q' \\
  \{ P' \} \ S \ \{ Q' \}
  \]

- **An inference rule for sequences**
  For a sequence \( S_1; S_2 \):
  \[
  \{ P_1 \} \ S_1 \ \{ P_2 \} \quad \{ P_2 \} \ S_2 \ \{ P_3 \}
  \]
  the inference rule is:
  \[
  \{ P_1 \} \ S_1; S_2 \ \{ P_3 \}
  \]

Conditions

Here’s a rule for a conditional statement

\[
\{ B \land P \} \ S_1 \ \{ Q \}, \quad \{ \neg B \land P \} \ S_2 \ \{ Q \}
\]

The postcondition of \( S_1 \) and \( S_2 \) is \( Q \).

And an example of it’s use for the statement

\[
\{ P \} \ \text{if } x > 0 \ \text{ then } y = y - 1 \ \text{ else } y = y + 1 \ \{ y > 0 \}
\]

So the weakest precondition \( P \) can be deduced as follows:

- The postcondition of \( S_1 \) and \( S_2 \) is \( Q \).
- The weakest precondition of \( S_1 \) is \( x > 0 \land y < 1 \) and for \( S_2 \) is \( x > 0 \land y > 1 \)
- The rule of consequence and the fact that \( y > 1 \Rightarrow y > -1 \) supports the conclusion
- That the weakest precondition for the entire conditional is \( y > 1 \).

Loops

For the loop construct \( \{ P \} \ \text{while } B \ \text{do } S \ \text{end} \ \{ Q \} \)
the inference rule is:

\[
\{ I \land B \} \ S \ \{ I \} \quad \{ I \} \ \text{while } B \ \text{do } S \ \{ I \land \neg B \}
\]

where \( I \) is the loop invariant, a proposition necessarily true throughout the loop’s execution.
Loop Invariants

A loop invariant $I$ must meet the following conditions:

1. $P \implies I$ (the loop invariant must be true initially)
2. $\{I\} B \{I\}$ (evaluation of the Boolean must not change the validity of $I$)
3. $\{I \land B\} S \{I\}$ (I is not changed by executing the body of the loop)
4. $(I \land \neg B) \implies Q$ (if $I$ is true and $B$ is false, $Q$ is implied)
5. The loop terminates (this can be difficult to prove)

- The loop invariant $I$ is a weakened version of the loop postcondition, and it is also a precondition.
- I must be weak enough to be satisfied prior to the beginning of the loop, but when combined with the loop exit condition, it must be strong enough to force the truth of the postcondition.

Evaluation of Axiomatic Semantics

- Developing axioms or inference rules for all of the statements in a language is difficult.
- It is a good tool for correctness proofs, and an excellent framework for reasoning about programs.
- It is much less useful for language users and compiler writers.

Denotational Semantics

- A technique for describing the meaning of programs in terms of mathematical functions on programs and program components.
- Programs are translated into functions about which properties can be proved using the standard mathematical theory of functions, and especially domain theory.
- Originally developed by Scott and Strachey (1970) and based on recursive function theory.
- The most abstract semantics description method.

Denotational Semantics

- The process of building a denotational specification for a language:
  1. Define a mathematical object for each language entity.
  2. Define a function that maps instances of the language entities onto instances of the corresponding mathematical objects.
- The meaning of language constructs are defined by only the values of the program's variables.
**Denotational Semantics (continued)**

The difference between denotational and operational semantics: In operational semantics, the state changes are defined by coded algorithms; in denotational semantics, they are defined by rigorous mathematical functions.

- The state of a program is the values of all its current variables:
  \[ s = \{ <i_1, v_1>, <i_2, v_2>, \ldots, <i_n, v_n> \} \]

- Let VARMAP be a function that, when given a variable name and a state, returns the current value of the variable:
  \[ \text{VARMAP}(i_j, s) = v_j \]

**Example: Decimal Numbers**

<table>
<thead>
<tr>
<th>&lt;dec_num&gt;</th>
<th>( 0 )</th>
<th>( 1 )</th>
<th>( 2 )</th>
<th>( 3 )</th>
<th>( 4 )</th>
<th>( 5 )</th>
<th>( 6 )</th>
<th>( 7 )</th>
<th>( 8 )</th>
<th>( 9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle \text{dec_num} \rangle )</td>
<td>( 0 )</td>
<td>( 1 )</td>
<td>( 2 )</td>
<td>( 3 )</td>
<td>( 4 )</td>
<td>( 5 )</td>
<td>( 6 )</td>
<td>( 7 )</td>
<td>( 8 )</td>
<td>( 9 )</td>
</tr>
</tbody>
</table>

\[
M_{\text{dec}}('0') = 0, \quad M_{\text{dec}}('1') = 1, \quad \ldots, \quad M_{\text{dec}}('9') = 9
\]

\[
M_{\text{dec}}(<\text{dec_num}> '0') = 10 \cdot M_{\text{dec}}(<\text{dec_num}>)
\]

\[
M_{\text{dec}}(<\text{dec_num}> '1') = 10 \cdot M_{\text{dec}}(<\text{dec_num}>) + 1
\]

\[
\ldots
\]

\[
M_{\text{dec}}(<\text{dec_num}> '9') = 10 \cdot M_{\text{dec}}(<\text{dec_num}>) + 9
\]

**Expressions**

\[
M_{\text{e}}(<\text{expr}>), s) \Delta =
\text{case } <\text{expr}> \text{ of}
\langle \text{dec_num} \rangle \Rightarrow M_{\text{dec}}(<\text{dec_num}>), s)
\langle \text{var} \rangle \Rightarrow
\text{if } \text{VARMAP}(<\text{var}>, s) = \text{undef}
\text{then error}
\text{else } \text{VARMAP}(<\text{var}>, s)
\langle \text{binary_expr} \rangle \Rightarrow
\text{if } (M_{\text{e}}(<\text{binary_expr}>,<\text{left_expr}>), s) = \text{undef}
\text{OR } M_{\text{e}}(<\text{binary_expr}>,<\text{right_expr}>), s) = \text{undef}
\text{then error}
\text{else}
\text{if } (<\text{binary_expr}>,<\text{operator}>) = '+' \text{ then}
\text{else } M_{\text{e}}(<\text{binary_expr}>,<\text{left_expr}>), s) +
\text{else } M_{\text{e}}(<\text{binary_expr}>,<\text{left_expr}>), s) *
\]

**Assignment Statements**

\[
M_{\text{a}}(x := E, s) \Delta =
\text{if } M_{\text{e}}(E, s) = \text{error}
\text{then error}
\text{else } s' = \{ <i_1',v_1'>, <i_2',v_2'>, \ldots, <i_n',v_n'> \},
\text{where for } j = 1, 2, \ldots, n,
\text{if } i_j \neq x \text{ then } v_j' = \text{VARMAP}(<i_j,s>) \text{ if } i_j \leftrightarrow x
\text{else } M_{\text{e}}(E, s) \text{ if } i_j = x
Logical Pretest Loops

\[ M_l(\text{while } B \text{ do } L, s) \Delta= \]

\[
\begin{align*}
\text{if } M_b(B, s) &= \text{undef} \\
\text{then error} \\
\text{else if } M_b(B, s) &= \text{false} \\
\text{then } s \\
\text{else if } M_{sl}(L, s) &= \text{error} \\
\text{then error} \\
\text{else } M_l(\text{while } B \text{ do } L, M_{sl}(L, s))
\end{align*}
\]

Logical Pretest Loops

- The meaning of the loop is the value of the program variables after the statements in the loop have been executed the prescribed number of times, assuming there have been no errors
- In essence, the loop has been converted from iteration to recursion, where the recursive control is mathematically defined by other recursive state mapping functions
- Recursion, when compared to iteration, is easier to describe with mathematical rigor

Denotational Semantics

*Evaluation of denotational semantics:*

- Can be used to prove the correctness of programs
- Provides a rigorous way to think about programs
- Can be an aid to language design
- Has been used in compiler generation systems

Summary

This chapter covered the following

- Backus-Naur Form and Context Free Grammars
- Syntax Graphs and Attribute Grammars
- Semantic Descriptions: Operational, Axiomatic and Denotational